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OUR NOTIONS

OF

NUMBER AND SPACE

BY

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INTRODUCTION.

My thesis I briefly state as follows: Our brain habits, with the modes of thought and of judgment dependent thereon, are morphological resultants of definite past experiences: our experiences, and those of our ancestors. Each limited experience does its share toward fixing a limited habit. The experiences most common to our various regions of skin, differ widely one from another; those of the tongue, from those of the fingers; those of the fingers, from those of the abdomen, and so on. Our habits of judgment, based on these several avenues of experience, ought therefore, when compared with each other, to betray permanent characteristics running parallel with the local differences of anatomy, of function, and of experience, which give rise to them, and in which they are rooted.

Investigation proves this to be the case. It shows that our judgments of the same outer facts, such as of number and of distance, vary greatly when mediated by different tactual regions. And what is of greater importance to the science of psychology, these variations in judgment bear distinguishing ear-marks of the

kinds of experience out of which, and by reason of which through life, they have slowly risen.

It is our purpose to study these. Through comparison of the different constants and variables in certain judgments, which we shall subject to experimental proof and analysis, we aim to discover somewhat regarding the fundamental laws governing the past genesis and the present formation of our judgments, and of the movements of mental processes in general.

As the last words of this Introduction, I wish to thank Professor Münsterberg for permitting one of his students to assist me with this research during an entire year. And with deep appreciation, and pleasant recollections, I record the patient labor and able service which Mr. Parsons has continually contributed to the work.

OUR NOTIONS OF NUMBER AND SPACE.

EXPERIMENTS A, B, C, D, E, F, G, and H.

THESE several experiments form a set. We shall first present the method, and the bare results of each one separately, then study them collectively.

EXPERIMENT A.

WITH PINS SET IN A STRAIGHT LINE.

Apparatus. — Heavy cardboard was cut in strips 7 or 8 mm. narrower than the pins to be used. The pins were the familiar household article; they were run through the whole width of the strip, which held them firmly, their ends projecting like the teeth of a comb.

Thirty-six cards were thus prepared, or 9 sets of 4 cards each. The "9 sets" corresponded to the 9 distances experimented with; and by "distance" we shall always denote the distance between the end pins of the line of pins. The 9 distances embraced the even and the half cmm. from 1 to 5 inclusive.

The 4 cards of each "distance set" were fitted with 2, 3, 4, and 5 pins respectively. These pins (when

more than 2) were spaced equally apart for each card; these sub-distances, of course, varying on each separate card.

Thus prepared, our 36 cards represented 9 categories of "distance," and 4 categories of "number" for each distance. (For the abdomen an extra 6-pin card was used.)

A holder was provided for these cards, in order that the subject, when taking them in his hand, should learn nothing about them through the pinch of his fingers. This holder was a folded strip of sheet-steel, — the cards were dropped into its groove, with the heads of the pins resting against the metal, and the sides of the holder were then pinched together.

Method. — In this experiment the subject applied the pins to himself, the cards being drawn, put into the holder, and handed to him by some one else. In applying the cards the subject was permitted to rock the pins back and forth on his skin. The line of direction, in which the line of pins were applied, was for each locality always the same. This was at right angles to the median line on the tongue, the forehead, and the abdomen, and longitudinally on the forearm.

At first an instrument was used to regulate the pressure with which the pins were applied; but it was soon found, the pins being sharp, that the subject's own feeling, adapting itself very sensitively to the conditions

of best judgment under varying conditions of thickness and toughness of skin, was a better "control" for the "constancy of pressure" than any mechanical contrivance could possibly be.

Proper care was used to avoid complications due to fatigue or to changes of temperature.

Explanations of the Tables. — In this experiment: A four persons were experimented upon as follows. B, a student of biology; L, a student of psychology; P, Mr. Parsons; and N, myself. Each card of pins was applied 100 times to each person. The "distance" categories are indicated in the left-hand vertical column and govern the horizontal line of figures opposite to them, across the page. The "number" categories, showing the number of pins in each card, are indicated by Roman numerals in the top horizontal heading, and govern the vertical column of figures below, to the bottom of the page. The main body of figures shows averages calculated from 100 applications of each card to each of the four persons, *i.e.*, from a total of 400 applications.

The four main horizontal divisions of the tables show as follow: —

The First: — Shows the number of times, per hundred times applied, that the number of pins was judged correctly.

The Second: — Shows the per cent. error made in judging the number of pins.

The Third: — Shows the number of times per hundred times applied, that the distance was judged correctly.

The Fourth: — Shows the per cent. error made in judging the distance.

Tables 1, 2, 3, and 4 are wholly "regular" according to the foregoing explanations.

Table 5. — The question arose as to what part the rocking of the pins back and forth on the skin, which was permitted the subject, played in making his judgment. Or to put the matter more psychologically, the question rose as to how far such judgments were direct, and how far complex and reasoned out. To throw light on this matter, a set of tests was made upon the forearm, which differed from the regular experiments in that the pins were only permitted to be pressed upon the skin steadily and evenly throughout the whole line, and but three times in regular succession, at intervals one second apart. The results so obtained are reported in Table 5.

Table 6. — Particularly it was suspected, that our judgments of the number of pins in a given card were reasoned out somewhat as follows: that, feeling perhaps, the two end pins widely apart, and two intermediate pins nearer together, or even one pin at some intermediate point, we then said: — "since the intermediate pins are spaced equally there ought to be so many pins"; thus arriving at the final estimate by

mathematical calculations based on the partial data actually given in the impression.

It was found that, with practice, this sort of reckoning process could be largely suppressed by volition; by concentrating our attention upon the number of points felt, and strenuously shutting out all else from the impression. If we could not succeed in this perfectly, it was well to compare results obtained by this method, with those where, as in the regular experiments, every possible aid was given to forming the judgments. Table 6 presents such results.

Table 7.—It being a main proposition of this research to compare results obtained upon dissimilar regions of the body, it was requisite that all the results should be obtained under conditions as similar as possible. As a matter of fact the different regions were for each separate experiment worked upon successively, and the full set of tests was finished for one region before proceeding to another. At the end, the question arose: how were the later results influenced by the considerable amount of skill and practice acquired in the foregoing work? To test this, still another series, perfectly regular in its method, was taken upon the forearm, at the very end of all our work. Table 7 contains its results. The regions were first worked on in the following order, *i.e.*, tongue, forehead, fore-

arm, and abdomen, and the corresponding tables are arranged in similar order.

Tables 8 and 9. — For theoretical reasons to be reached in our future discussions, it became desirable to have, for comparison with our other results, a set of judgments from impressions where the distance categories remained the same as in our regular experiments, but where the number of pins, or points, in each line should be increased to a maximum, or to infinity. That is, wherein a straight line, or straight edge should be pressed upon the skin, instead of pins set at intervals in a line. Accordingly a series of tests was made upon the forearm, with a set of cards, cut to proper lengths from thin, hard card-board, the whole length of the edge of the card being pressed directly upon the skin. Table 8 shows results obtained with them, according to the regular method of permitting the subject to "rock" the card upon the skin. Table 9 shows results comparative with those of Table 5, where the cards were pressed evenly and steadily three times in succession, at intervals one second apart.

Table 10 is a general summary of the foregoing tables, and aids in comparing the different regions worked upon.

NOTE. — A glance at any of our tables shows them divided into sub-tables, or blocks. Each block bears a number, in parenthesis, in its upper middle portion. I shall always refer to these minor

tables as "blocks," identifying them by their proper numbers. The total number of blocks is 356. But, owing to the great expense that would be incurred, I am unable to publish the results obtained from the individual subjects, and can lay before the present reader only the blocks and figures which present the averages calculated from the four subjects. The original figures, however, are at the service of any one who cares for them.

Table 1. Experiment A. — Pins in straight line.

TONGUE.

Subject		Distance between End Pins	Average of N. P. L. and B.				
No. Pins in Line		(Centimeters)	II	III	IV	V	Averages
Judgments of Number of Pins.	No. times judged correctly per 100 times applied.	(5)					
		1	100	97	96	99	
		1.5	100	99	98	99	
		2	100	100	97	98	
		2.5	100	100	98	97	
		3	100	99	97	96	
		Averages	100	99	97.3	97.8	98.5
	Per cent. of Error of No. Pins judged.	(10)					
		1	0	+1.1	+1.2	− .3	
		1.5	0	+ .3	+ .5	− .2	
		2	0	0	+ .3	− .4	
		2.5	0	0	+ .5	− .2	
		3	0	+ .3	− .2	− .8	
		Averages	0	+ .3	+ .5	− .4	+.075
Judgments of Distance.	No. times judged correctly per 100 times applied.	(15)					
		1	98	99	97	100	98.5
		1.5	98	91	91	97	94.2
		2	95	94	95	97	95.2
		2.5	91	95	96	98	95.0
		3	98	94	93	97	95.5
		Averages	96.0	94.6	94.4	97.8	95.68
	Per cent. of Error of Distance judged.	(20)					
		1	+1.0	+ .5	+1.5	0	+ .7
		1.5	+ .7	+3.0	+2.5	+1.3	+1.9
		2	+1.2	+1.2	+ .1	− .4	+ .5
		2.5	+1.4	+ .4	+ .6	+ .2	+ .6
		3	− .3	−1.0	−1.2	− .5	− .7
		Averages	+ .8	+ .8	+ .7	+ .1	+ .6

Table 2. Experiment A. — Pins in straight line.

FOREHEAD.

Subject / No. Pins in Line		Distance between End Pins (Centimeters)	Average of N. P. L. and B. — II	III	IV	V	Averages
Judgments of Number of Pins.	No. times judged correctly per 100 times applied.		(25)				
		1	5	16	61	64	
		1.5	28	27	76	41	
		2	58	8	60	30	
		2.5	96	32	57	26	
		3	99	60	52	26	
		Averages	57.2	28.6	61.2	37.2	46.0
	Per cent. of Error of No. Pins judged.		(30)				
		1	+84.2	+43.5	+1.5	− 8.7	
		1.5	+58.8	+24.8	−9.0	−14.9	
		2	+30.5	+25.1	−8.3	−17.8	
		2.5	+ 1.5	+22.8	−2.0	−20.0	
		3	+ .2	+ 4.3	−8.0	−22.0	
		Averages	+35.0	+24.0	−5.2	−16.7	+9.275
Judgments of Distance.	No. times judged correctly per 100 times applied.		(35)				
		1	30	65	78	90	65.7
		1.5	57	39	43	36	43.7
		2	51	47	45	40	45.7
		2.5	68	49	51	48	54.0
		3	82	78	60	56	69.0
		Averages	57.6	55.1	55.4	54.0	55.62
	Per cent. of Error of Distance judged.		(40)				
		1	+42.1	+16.4	+11.2	+ 7.0	+19.2
		1.5	+ 4.7	+ 6.7	+ 1.2	− 9.9	+ .7
		2	+ 1.0	+ 1.1	− 9.3	−11.2	− 4.6
		2.5	+ 3.6	− 6.8	− 7.3	−12.3	− 5.7
		3	− 3.2	− 6.1	−11.1	−14.1	− 8.6
		Averages	+ 9.6	+ 2.5	− 2.5	− 8.1	+ 2.0

Table 3. Experiment A. — Pins in straight line.

FOREARM.

Subject		Distance between End Pins	Average of N. P. L. and B.				
No. Pins in Line		(Centimeters)	II	III	IV	V	Averages
Judgments of Number of Pins.	No. times judged correctly per 100 times applied.				(45)		
		1	26	31	45	23	
		1.5	39	34	37	19	
		2	54	37	31	20	
		2.5	59	39	41	13	
		3	64	27	30	14	
		Averages	48.4	33.6	36.8	17.8	34.2
	Per cent. of Error of No. Pins judged.				(50)		
		1	+70.0	+18.1	−11.7	−27.3	
		1.5	+47.0	+ 9.3	−14.2	−27.7	
		2	+43.0	+ 7.1	−15.1	−23.9	
		2.5	+38.0	+ .3	−18.3	−29.9	
		3	+29.0	+ .9	−21.1	−29.1	
		Averages	+45.4	+ 7.2	−16.1	−27.6	+2.225
Judgments of Distance.	No. times judged correctly per 100 times applied.				(55)		
		1	20	35	42	46	35.7
		1.5	47	33	29	27	44.0
		2	55	50	44	40	47.2
		2.5	41	41	39	40	40.2
		3	57	40	49	48	48.5
		Averages	44.0	39.8	40.4	40.2	43.16
	Per cent. of Error of Distance judged.				(60)		
		1	+69.1	+56.0	+44.3	+40.0	+52.4
		1.5	+33.1	+29.4	+25.1	+21.0	+27.1
		2	− .2	+ .5	− 1.6	+ .9	− .1
		2.5	− 4.3	− 6.9	− 7.3	− 9.0	− 6.9
		3	− 9.1	−12.3	−14.9	−15.1	−12.8
		Averages	+17.7	+13.3	+ 9.1	+ 7.6	+11.9

Table 4a. Experiment A. — Pins in straight line.

ABDOMEN.

Subject / No. Pins in Line	Distance between End Pins (Centimeters)	II	III	IV	V	VI	Averages
		Averages of N. P. L. and B.					
Judgments of Number of Pins. — No. times judged correctly per 100 times applied.				(65)			
	1	7	32	59	55		
	1.5	8	21	57	52		
	2	11	16	55	47		
	2.5	19	20	51	42		
	3	29	28	44	44		
	3.5	34	29	44	37		
	4	52	30	49	35		
	4.5	68	36	41	26		
	5	79	38	36	28	14	
	Averages	34.1	27.8	48.4	40.7	14.0	33.0
Judgments of Number of Pins. — Per cent. of Error of No. Pins judged.				(70)			
	1	+109.9	+34.1	+12.1	−13.3		
	1.5	+ 99.4	+39.8	+12.4	−13.9		
	2	+ 90.1	+38.7	+10.1	−14.9		
	2.5	+ 78.8	+37.2	+ 9.1	−16.2		
	3	+ 62.3	+29.1	+ 9.1	−17.4		
	3.5	+ 53.9	+30.1	+ 6.3	−18.2		
	4	+ 42.4	+25.1	+ 4.2	−21.4		
	4.5	+ 31.2	+11.4	− 6.1	−24.9		
	5	+ 21.2	+ .2	−12.1	−25.5	−22.2	
	Averages	+ 6	+27.3	+ 5.2	−18.4	−22.2	+11.44

Table 4b. Experiment A. — Pins in straight line.

ABDOMEN.

	Subject / No. Pins in Line	Distance between End Pins (Centimeters)	II	III	IV	V	VI	Averages
Judgments of Distance.	No. times judged correctly per 100 times applied.				(75)			
		1	42	53	66	78		59.7
		1.5	25	24	31	30		27.5
		2	28	29	28	24		27.2
		2.5	28	28	26	25		26.7
		3	28	27	27	25		26.7
		3.5	23	21	18	20		20.5
		4	27	25	22	23		24.2
		4.5	26	26	24	20		24.0
		5	48	44	40	39	38	40.2
		Averages	30.6	30.8	31.3	31.6	38.0	30.74
	Per cent. of Error of Distance judged.				(80)			
		1	+37.0	+29.8	+21.0	+16.1		+26.0
		1.5	+ 8.9	+ 8.4	+ 3.3	— 8.1		+ 3.1
		2	+12.8	+ 9.9	+ 1.0	— 1.2		+ 5.6
		2.5	+12.3	+ 1.1	— 3.1	— 6.2		+ 1.0
		3	+13.2	+ 9.2	+ .5	— .3		+ 5.6
		3.5	+12.0	+ 8.2	+ .6	— .4		+ 5.1
		4	+ 8.9	+ .4	— .4	— .4		+ 2.1
		4.5	— .8	— .5	— 4.2	— 7.7		— 3.3
		5	—11.7	—12.5	—14.7	—15.5	—16.2	—14.1
		Averages	+10.3	+ 6.0	+ .4	— 2.6	—16.2	+ 3.46

Table 5. Experiment A. — Pins in straight line.

(a) FOREARM. Pressing evenly, three times only.

Subject		Distance between End Pins	Average of N. and P.				
No. Pins in Line		(Centimeters)	II	III	IV	V	Averages
Judgments of Number of Pins.	No. times judged correctly per 100 times applied.		(83)				
		1	13	21	35	33	
		1.5	14	22	26	34	
		2	26	17	29	40	
		2.5	20	22	34	32	
		3	41	21	40	32	
		Averages	22.8	20.6	32.8	34.2	27.6
	Per cent. of Error of No. Pins judged.		(86)				
		1	+78.0	+26.0	− 5.2	−17.6	
		1.5	+79.5	+26.7	− .8	−16.4	
		2	+75.5	+28.5	− 4.2	−14.4	
		2.5	+71.5	+21.2	− 4.1	−18.0	
		3	+66.5	+21.0	− 4.5	−16.7	
		Averages	+74.2	+24.7	− 3.8	−16.6	+19.625
Judgments of Distance.	No. times judged correctly per 100 times applied.		(89)				
		1	12	6	13	15	11.5
		1.5	28	16	16	12	18.0
		2	29	25	32	25	27.7
		2.5	42	33	25	26	31.5
		3	38	41	40	40	39.7
		Averages	29.8	24.2	25.2	23.6	25.86
	Per cent. of Error of Distances judged.		(92)				
		1	+86.5	+104.0	+106.0	+98.5	+98.7
		1.5	+29.5	+ 41.1	+ 29.8	+55.8	+38.8
		2	+ 1.6	+ 2.8	+ 6.5	+ 9.9	+ 5.2
		2.5	− 2.2	− 2.7	− 1.1	− 2.5	− 2.1
		3	−13.4	− 13.6	− 13.5	−15.6	−14.0
		Averages	+20.4	+ 26.3	+ 25.6	+19.2	+11.94

Table 6. Experiment A. — Pins in straight line.

(b) FOREARM.

Attention given only to number points felt.

SUBJECT / No. Pins in Line		Distance between End Pins (Centimeters)	II	III	IV	V	Averages
Judgments of Number of Pins.	No. times judged correctly per 100 times applied.			(95)			
		1	84	21	11	0	
		1.5	81	25	7	1	
		2	79	27	14	0	
		2.5	74	23	27	5	
		3	79	24	27	2	
		Averages	79.4	24.0	17.2	1.4	30.5
	Per cent. of Error of No. Pins judged.			(98)			
		1	+23.1	+12.4	−27.0	−43.6	
		1.5	+19.8	+13.0	−27.2	−48.9	
		2	+14.4	+14.2	−32.9	−47.2	
		2.5	+19.5	+16.9	−33.5	−47.0	
		3	+14.8	+14.7	−26.6	−53.1	
		Averages	+18.3	+14.2	−29.4	−48.0	−11.225
Number of times the judgment "two pins" was made per 100 times applied.				(101)			
		1	84	62	55	51	
		1.5	81	54	56	58	
		2	79	60	60	48	
		2.5	74	63	42	53	
		3	79	54	37	52	
		Averages	79.4	58.6	50.0	52.4	

Table 7. Experiment A. — Pins in straight line.

(c) FOREARM. Test of improvement through practice.

	SUBJECT / No. Pins in Line	Distance between End Pins (Centimeters)	AVERAGE OF N. AND P. II	III	IV	V	Averages
JUDGMENTS OF NUMBER OF PINS.	No. times judged correctly per 100 times applied.				(104)		
		1	12	30	50	28	
		1.5	26	24	56	26	
		2	38	34	48	12	
		2.5	50	28	52	12	
		3	64	36	48	14	
		Averages	38.0	30.4	50.8	18.4	34.4
	Per cent. of Error of No. of Pins judged.				(107)		
		1	+84.0	+22.6	− 8.0	−26.0	
		1.5	+64.0	+18.0	−11.0	−19.6	
		2	+50.0	+21.3	−10.5	−23.6	
		2.5	+40.0	+ 8.0	− 9.0	−30.0	
		3	+24.0	+ .3	− 8.0	−26.4	
		Averages	+52.4	+14.0	− 9.3	−25.1	+8.0
JUDGMENTS OF DISTANCE.	No. times judged correctly per 100 times applied.				(110)		
		1	36	24	30	28	29.5
		1.5	32	30	34	36	33.0
		2	28	50	42	38	39.5
		2.5	40	32	30	46	37.0
		3	84	86	74	68	78.0
		Averages	44.0	44.4	42.0	43.2	43.4
	Per cent. of Error of Distance judged.				(113)		
		1	+48.0	+ 60.0	+ 55.0	+53.0	+54.0
		1.5	+25.3	+ 26.6	+ 22.0	+21.3	+23.8
		2	+20.0	+ 9.5	+ 9.0	+ 7.0	+11.4
		2.5	+ 3.6	− 2.0	− 4.8	− 1.6	− 1.2
		3	− 2.6	− 2.3	− 11.3	− 5.6	− 5.4
		Averages	+18.9	+ 19.2	+ 12.2	+14.8	+16.5

Tables 8 and 9. Experiment A. — Supplement.

STRAIGHT-EDGE.

FOREARM.

		(8) (d) Regular.		(9) (e) Pressing evenly three times only.
		Distance or Length of Straight-Edge	Average of N. P. L. and B.	Average of N. P. L. and B.
Judgments of Distance.	No. times judged correctly per 100 times applied.		(114)	(116)
		1	53.0	48.7
		1.5	50.0	39.7
		2	60.0	46.5
		2.5	72.0	44.5
		3	83.0	55.5
		Averages	63.8	47.0
	Per cent. of Error of Distance judged.		(115)	(117)
		1	+31.4	+ 34.8
		1.5	+ 8.9	+ 10.6
		2	+ 2.7	+ 1.3
		2.5	− .6	− .8
		3	− 7.8	− 1.5
		Averages	+ 6.9	+ 8.9

Table 10a. Experiment A. — Pins in straight line.

SUMMARY. — Averages brought forward from foregoing tables.

	Region.	General Averages from Whole Set of Distances (1 to 5 cmtrs.) Worked on, and from the Four Persons (N. P. L. and B.).											
		Judgments of Number of Pins.						Judgments of Distances.					
	No. Pins in Line.	II	III	IV	V	VI	Total Averages.	II	III	IV	V	VI	Total Averages.
No. times judged correctly per 100 times applied.	Tongue	100.0	99.0	97.2	97.8		98.5	96.0	94.6	94.4	97.8		95·68
	Forehead	57.2	28.6	61.2	37.0		46.0	57.6	55.1	55.4	54.0		55.62
	Forearm	50.4	33.6	36.8	17.8		34.6	44.0	39.8	40.4	40.2		43.16
	Abdomen	34.1	27.3	48.4	41.0	14.0	33.0	30.6	30.8	31.3	31.6	38.0	30.74
	Forearm (*a*)	22.8	20.6	32.0	34.2		27.6	29.8	24.2	25.2	23.6		25.86
	Forearm (*b*)	79.4	24.0	17.2	1.4		30.5						
	Forearm (*c*)	38.0	30.4	50.8	18.4		34.4	44.0	44.4	42.0	43.2		43.40
	Forearm (*d*)												163.6
	Forearm (*e*)												47.0
	Total Averages	54.6	37.7	49.2	35.4	14.0	43.2	50.3	48.1	48.1	48.4	38.0	48.32*
Per cent. of Error.	Tongue		+ .3	+ .4	− .4		+ .075	+ .8	+ .8	+ .7	+ .1		+ .60
	Forehead	+35.0	+24.0	− 5.2	−16.7		+ 9.275	+ 9.6	+ 2.5	− 2.5	+ 8.1		+ .20
	Forearm	+45.4	+ 7.2	−16.1	−27.6		+ 2.225	+17.7	+13.3	+ 9.1	+ 7.6		+11.90
	Abdomen	+65.5	+27.3	+ 5.0	−18.4	−22.2	+11.440	+10.3	+ 6.0	+ .4	− 2.6	−16.2	+ 3.46
	Forearm (*a*)	+74.2	+24.7	− 3.8	−16.6		+19.625	+20.4	+26.3	+25.6	+19.2		+11.94
	Forearm (*b*)	+18.3	+14.2	−29.4	−48.0		−11.225						
	Forearm (*c*)	+52.4	+14.0	− 9.3	−25.1		+ 8.000	+18.9	+19.2	+12.2	+14.8		+16.50
	Forearm (*d*)												+ 6.90
	Forearm (*e*)												+ 8.90
	Total Averages	+41.6	+15.0	− 8.3	−21.8	−22.2	+ 5.831	+12.9	+11.3	+ 7.6	+ 7.9	−16.2	+10.60*

* The "Straight-edge" figures are not included in these totals.

Table 10b. Experiment A. — Pins in straight line.

SUMMARY. — Averages brought forward from foregoing tables, 1 to 9, inclusive.

	Region.	General Averages from the Various Numbers of Pins (II–VI) Worked with, and from the Four Persons (N. P. L. and B.). Judgments of Distances.									
	Distances between end pins (cmtrs.)	1	1.5	2	2.5	3	3.5	4	4.5	5	Total Averages.
No. times judged correctly per 100 times applied.	Tongue	98.5	94.2	95.2	95.0	95.5					95.68
	Forehead	65.7	43.7	45.7	54.0	69.0					55.62
	Forearm	35.5	44.0	47.5	40.2	48.5					43.16
	Abdomen	59.7	27.5	27.2	26.7	26.7	20.5	24.2	24.0	40.2	30.74
	Forearm (*a*)	11.5	18.0	27.7	31.5	39.7					25.86
	Forearm (*b*)										
	Forearm (*c*)	29.5	33.0	39.5	37.0	78.0					43.4
	Forearm (*d*)	53.0	50.0	60.0	72.0	83.0					63.6
	Forearm (*e*)	48.7	39.7	46.5	44.5	55.5					47.0
	Total Averages	50.1	43.4	47.1	47.4	59.6					48.32*
Per cent. of Error.	Tongue	+ .7	+ 1.9	+ .5	+ .6	− .7					+ .60
	Forehead	+19.2	+ .7	− 4.6	− 5.7	− 8.6					+ .20
	Forearm	+52.4	+27.1	− .1	− 6.9	−12.8					+11.90
	Abdomen	+26.0	+ 3.1	+ 5.6	+ 1.0	+ 5.6	+ 5.1	+ 2.1	− 3.3	−14.1	+ 3.46
	Forearm (*a*)	+98.7	+38.8	+ 5.2	− 2.1	−14.0					+11.94
	Forearm (*b*)										
	Forearm (*c*)	+54.0	+23.8	+11.4	− 1.2	− 5.4					+16.50
	Forearm (*d*)	+31.4	+ 8.9	+ 2.7	− .6	− 7.8					+ 6.90
	Forearm (*e*)	+34.8	+10.6	+ 1.3	− .8	− 1.5					+ 8.90
	Total Averages	+41.8	+15.9	+ 3.0	− 2.4	− 6.0					+10.60*

* The "Straight-edge" figures are not included in these totals.

EXPERIMENT B.

WITH PINS SET IN TRIANGLES AND SQUARES.

Apparatus. — Triangles and squares of proper dimensions were cut from specially heavy "trunk" cardboard. The pins were thrust through the board at right angles to its surface, care being taken to have their points lie perfectly in the same plane.

To the "distance" and "number" categories of Experiment A was now added the category of "figure." The two "figure" categories used in the present experiment were those of the triangle and the square.

The same "distance" categories were used as before in A; and, as before, these measured the distances between the end pins, or, as it would be in this case, measured the outside lines of the triangles and squares.

The "number" categories here used will easily be understood, while referring to the horizontal headings of the tables, if I explain that "III T" means a triangle with a pin in each corner; "IV T," a triangle with a pin in each corner and one in the center of the triangle; "VI T," a triangle with a pin in each corner and one bisecting each of the three sides; "VII T," a triangle with six pins arranged as above and still another pin in

the center. "IV S," "V S," "VIII S," and "IX S" indicate squares similarly arranged to the triangles.

Method. — This differed from that of Experiment A only in that the pins were now applied to the subject by some one other than himself, he not being able to handle this apparatus without learning thereby somewhat of the size of the board which held the pins.

The subject now had three judgments to make for every application. He usually made, and always announced these, in the same order, and as follows, *i.e.*, "distance," "number of pins," "figure."

Explanation of the Tables. — These tables are much like those of Experiment A, except that a fifth main horizontal division has been added, giving the number of correct judgments as to the figure in which the pins were arranged, calculated from one hundred applications to each person.

Also for the better comparison of the results from the "triangles" with those from the "squares," a more complicated arrangement of "averages" in the several minor or sub-tables was requisite. This, however, will be clear if, referring to Table 11, I explain that any figures found in the vertical column marked "T." are averages of the foregoing figures, in the same horizontal line, to be found under the four vertical columns marked "Triangles"; and those under "S.," similarly, are averages for the foregoing figures under "Squares."

Tables 11, 12, 13, and *14* are "regular" in method and compare, respectively, with Tables 1, 2, 3, and 4 of Experiment A.

Table 15 is "irregular," in that the pins were permitted to be pressed only three times in succession, as in Table 5.

Table 16. Attention concentrated solely on number of points actually felt, as in Table 6.

Table 11. Experiment B.—Pins set in triangles and squares.

TONGUE.

Person.		Distance	Averages of N. P. L. and B.										
Figure.			Triangles.				Squares.				T.	S.	T. and S.
No. Pins.		(Centimeters).	III	IV	VI	VII	IV	V	VIII	IX	Averages.		Total Averages.
Judgments of Number of Pins.	No. times judged correctly per 100 times applied.		(128)										
		1	98	99	98		98	99	98				
		1.5	99	99	97		99	98	99				
		2	99	99	98		100	99	99				
		2.5	100	100	99	97	100	100	98	97			
		3	100	100	99	98	100	100	99	99			
		Averages	99.2	99.4	98.2	97.5	99.4	99.2	98.6	98.0	98.6	98.8	98.7
	Per cent. of Error of No. Pins judged.		(133)										
		1	+ .6	+ .2	− .3		+ .3	+ .2	− .3				
		1.5	+ .3	+ .2	− .5		+ .2	+ .4	− .1				
		2	+ .2	+ .1	− .3		0	+ .2	− .1				
		2.5	0	0	− .1	− .4	0	0	+ .1	− .3			
		3	0	0	+ .1	− .3	0	0	− .1	− .1			
		Averages	+ .2	+ .1	− .2	− .3	+ .1	+ .1	− .1	− .2	−.05	−.025	−.037

JUDGMENTS OF DISTANCE.	No. times judged correctly per 100 times applied.						(138)						
		1	97	98	99		98	99	98		98.	98.3	98.2
		1.5	98	96	96		98	97	97		96.7	97.3	97.0
		2	98	98	97		98	95	97		97.7	96.7	97.2
		2.5	100	100	98	97	100	100	98	99	98.7	99.2	99.0
		3	100	100	99	98	100	100	99	99	99.2	99.5	99.4
		Averages	98.6	98.4	97.8	97.5	98.8	98.2	97.8	99.0	98.1	98.9	98.5
	Per cent. of Error of Distance judged.						(143)						
		1	+1.3	+ .5	− .5		+1.0	+ .5	−1.0		+ .4	+ .1	+ .2
		1.5	+1.3	+1.3	−2.6		+1.3	+ .6	−1.9		0	0	0
		2	+1.2	+ .5	−1.5		+1.0	+1.5	−1.5		+ .1	+ .3	+ .2
		2.5	0	0	+ .3	−1.3	0	0	+ .4	− .3	− .2	0	− .1
		3	0	0	− .3	− .8	0	0	−1.5	− .4	− .3	− .5	− .4
		Averages	+ .8	− .5	− .9	−1.0	+ .7	+ .5	−1.1	− .3	− .1	0	0
JUDGMENTS OF FIGURE.	No. of correct judgments per 100 times applied.	NOTE. — As on the tongue the 'figure' could always be judged correctly, all the numbers of this division would be 100, and therefore need not be filled in.											

Table 12. Experiment B. — Pins set in triangles and squares.

FOREHEAD.

PERSON.		AVERAGES OF N. P. L. AND B.										
FIGURE.	DISTANCE (CENTIMETERS).	TRIANGLES.				SQUARES.				T.	S.	T. AND S.
NO. PINS.		III	IV	VI	VII	IV	V	VIII	IX	Averages.		Total Averages.
JUDGMENTS OF NUMBER OF PINS. No. times judged correctly per 100 times applied.						(148)						
	1	26	28	54		27	54	78				
	1.5	49	48	51		54	52	76				
	2	56	51	52		61	68	79				
	2.5	61	51	50	40	60	70	76	60			
	3	74	61	43	50	81	80	80	71			
	3.5	90	65	40	59	92	84	82	82			
	Averages	59.3	50.7	46.7	49.7	62.5	68.0	78.5	71.0	51.6	70.0	60.8
Per cent. of Error of No. Pins judged.						(153)						
	1	+69.1	+35.0	−12.9		+53.5	+28.9	− 8.9				
	1.5	+31.1	+11.1	−20.1		+20.1	+ 9.8	− 9.9				
	2	+29.8	+10.9	−21.0		+19.7	+ 9.1	−10.1				
	2.5	+21.0	+ 8.8	+ 8.7	−10.4	+13.7	+ 8.0	+ 1.0	− .9			
	3	+11.3	+ .6	+ 8.2	− 9.4	+11.0	+ .8	+ .5	− 4.2			
	3.5	+ 3.2	+ .1	+ 5.9	− 9.1	+ 3.0	− .9	+ .1	− 1.1			
	Averages	+27.6	+11.1	+ 5.2	− 9.6	+20.2	+ 9.3	− 4.5	− 1.0	+5.6	+6.0	+5.8

Judgments of Distance.	No. of correct judgments per 100 times applied.						(158)						
		1	87	88	91		78	83	90		88.7	83.7	86.2
		1.5	81	80	79		78	79	78		80.0	78.3	79.2
		2	82	70	62		77	69	71		71.3	72.3	71.8
		2.5	69	65	66	67	78	68	66	68	66.7	70.0	68.4
		3	71	71	70	72	76	74	75	75	71.0	75.0	73.0
		3.5	96	83	95	91	97	93	96	99	91.2	96.2	93.8
		Averages	81.0	76.2	77.2	76.7	80.7	77.7	79.3	80.7	77.8	79.6	78.7
	Per cent. of Error of Distance judged.						(163)						
		1	+4.4	+3.1	+ .8		+8.0	+7.9	+4.0		+2.8	+6.6	+4.7
		1.5	+ .6	− .4	− 6.7		+ .6	− .5	− .6		−2.2	− .2	−1.2
		2	+3.1	− .1	−11.0		−3.1	−3.2	− .5		−2.7	−2.3	−2.5
		2.5	+1.6	+ .2	+ .4	−1.8	+ .6	− .5	− .6	+1.8	+ .1	+ .4	+ .2
		3	− .4	− .8	+ .6	− .8	− .8	− .1	− .5	− .6	− .3	− .5	− .4
		3.5	− .6	−1.1	− .6	−1.5	− .9	− .8	− .6	− .7	−1.0	− .7	− .8
		Averages	+1.5	+ .1	− 2.7	−1.4	+ .7	+ .5	+ .2	+ .2	− .6	+ .4	− .1
Judgments of Figure.	No. of correct judgments per 100 times applied.						(168)						
		1	94	87	81		94	92	87		87.3	91.0	89.2
		1.5	97	95	94		98	98	98		95.3	98.0	96.7
		2	97	97	97		100	100	99		97.0	99.7	98.3
		2.5	100	99	100	100	100	100	100	100	99.7	100.0	99.9
		3	99	100	100	99	100	100	100	100	99.7	100.0	99.8
		3.5	100	100	99	100	100	100	100	100	99.7	100.0	99.9
		Averages	97.8	95.5	95.2	99.7	98.8	98.3	97.3	100	97.0	98.6	97.8

Table 13. Experiment B.—Pins set in triangles and squares.

FOREARM.

PERSON.		DISTANCE	AVERAGES OF N. P. L. AND B.										
FIGURE.			TRIANGLES.				SQUARES.				T.	S.	T. AND S.
NO. PINS.		(CENTIMETERS).	III	IV	VI	VII	IV	V	VIII	IX	Averages.		Total Averages.
JUDGMENTS OF NUMBER OF PINS.	No. times judged correctly per 100 times applied.						(173)						
		1	28	28	34		34	35	44				
		1.5	34	35	36		52	41	49				
		2	40	45	50		40	42	69				
		2.5	40	50	53	20	38	53	57	31			
		3	46	51	50	25	50	54	43	36			
		3.5	52	46	40	31	81	70	17	51			
		Averages	40.0	42.5	43.8	25.3	49.2	49.2	46.5	39.3	37.9	46.0	41.9
	Per cent. of Error of No. Pins judged.						(178)						
		1	+58.2	+21.4	− 9.5		+32.4	+10.3	−21.5				
		1.5	+45.0	+11.2	−18.1		+28.1	+10.0	−22.0				
		2	+28.6	+ 9.2	−15.0		+28.0	+ 9.8	−21.6				
		2.5	+20.2	+ 6.2	−13.0	−18.8	+27.9	+10.1	− 1.1	−18.4			
		3	+20.0	+ 8.5	− 4.0	−19.0	+19.8	+ 9.8	− .9	−11.0			
		3.5	+18.0	+ 8.4	− 4.0	−20.0	+ 7.0	+ 3.2	− 2.2	−11.1			
		Averages	+31.7	+10.8	−10.6	− 9.6	+23.9	+ 8.9	−11.5	−13.5	+5.6	+1.3	+3.4

JUDGMENTS OF DISTANCE.	No. times judged correctly per 100 times applied.						(183)						
		1	73	73	75		63	64	70		73.7	65.7	69.6
		1.5	69	73	75		78	79	72		72.3	76.3	74.3
		2	71	79	77		74	76	82		75.7	77.3	76.5
		2.5	70	69	71	59	78	69	77	60	67.2	71.0	69.2
		3	72	80	72	66	79	76	77	76	72.5	77.0	74.8
		3.5	97	89	96	92	98	94	98	92	93.5	95.5	94.5
		Averages	75.3	75.5	77.7	72.3	78.3	76.3	79.3	76.0	75.2	77.5	76.3
	Per cent. of Error of Distances judged.						(188)						
		1	+13.7	+11.1	+13.4		+21.0	+19.0	+15.0		+12.7	+18.3	+15.5
		1.5	+ .5	+ .2	− 2.1		+ .3	+ 1.0	+ 1.8		+ .9	+ 1.0	+ .9
		2	+ .9	+ .8	− 2.0		+ 1.4	+ 2.0	+ 2.0		+ 1.2	+ 1.8	+ 1.5
		2.5	+ .4	+ .9	+ 1.0	+4.2	+ 1.8	+ 3.0	+ .1	+3.1	+ 1.6	+ 2.0	+ 1.8
		3	+ .6	+ .8	− 1.1	−1.3	+ .9	− 1.1	+ .2	−1.8	− .3	+ .1	− .1
		3.5	− .2	− 1.0	− .4	−1.2	− .2	− .7	0	−2.0	− .7	− .7	− .7
		Averages	+ 2.6	+ 2.1	+ 1.5	+ .3	+ 4.2	+ 3.9	+ 3.2	− .2	+ 1.6	+ 2.8	+ 2.2
JUDGMENTS OF FIGURE.	No. times judged correctly per 100 times applied.						(193)						
		1	91	89	63		85	82	82		81.0	83.0	82.0
		1.5	82	86	90		97	95	97		86.0	96.3	91.2
		2	97	95	95		98	93	95		95.7	95.3	95.5
		2.5	98	97	98	98	99	95	99	99	97.7	98.0	97.9
		3	100	99	99	99	100	99	99	99	99.2	99.2	99.2
		3.5	96	95	97	98	100	96	99	99	96.5	98.5	97.5
		Averages	94.0	93.5	90.3	98.3	95.0	93.3	95.2	99.0	94.0	95.6	94.8

Table 14. Experiment B. — Pins set in triangles and squares.

ABDOMEN.

	Person.	Distance	Averages of N. and P.										
	Figure.		Triangles.				Squares.				T.	S.	T. and S.
	No. Pins.	(Centimeters).	III	IV	VI	VII	IV	V	VIII	IX	Averages.		Total Averages.
Judgments of Number of Pins.	No. times judged correctly per 100 times applied.							(196)					
		1	46	32	31		44	19	28				
		1.5	46	31	22		66	31	62				
		2	50	40	44		61	31	61				
		2.5	31	43	68	8	30	43	63	26			
		3	17	46	49	18	38	37	54	26			
		3.5	32	48	42	21	46	46	53	39			
		4	50	44	45	19	43	50	52	49			
		4.5	59	44	32	22	41	49	45	38			
		5	60	56	38	31	82	57	58	66			
		Averages	43.4	43.0	41.2	20.0	50.1	40.3	53.0	40.7	36.9	46.0	41.4
	Per cent. of Error of No. Pins judged.							(199)					
		1	+32.0	− 3.0	−19.3		+19.5	− 1.2	−30.0				
		1.5	+24.6	+ 2.5	−26.7		+14.5	+ 1.6	−16.2				
		2	+24.6	− 3.5	−19.3		+15.5	− 1.2	−18.0				
		2.5	+45.8	+ 9.5	−11.7	−25.4	+36.0	+ 5.2	− 7.7	−14.3			
		3	+54.0	+ 9.0	−13.3	−25.1	+43.0	+24.0	− 9.2	−15.3			
		3.5	+61.3	+16.5	− 4.0	−17.4	+24.5	+12.0	− 4.2	− 7.7			
		4	+22.0	+21.0	−12.1	−20.3	+33.0	+14.0	− 5.2	− 8.8			
		4.5	+25.3	+11.5	−13.1	−24.4	+43.5	+24.8	0	− 9.6			
		5	+13.3	+15.0	−14.0	−22.3	+ 4.5	+ 2.7	0	− 8.5			
		Averages	+33.9	+ 8.7	− 1.48	−22.5	+26.0	− 9.1	+10.1	−10.7	+1.3	+3.3	+2.3

JUDGMENTS OF DISTANCE.	No. times judged correctly per 100 times applied.							(202)					
		1	75	82	72		73	43	67		76.3	61.0	68.7
		1.5	59	57	55		60	73	65		57.0	66.0	61.5
		2	64	64	67		74	62	88		65.0	74.7	69.8
		2.5	59	66	58	61	74	72	76	68	61.0	72.5	66.8
		3	49	58	59	64	64	40	56	62	57.5	55.5	56.5
		3.5	60	45	51	62	51	46	41	59	54.5	49.2	51.9
		4	56	67	49	56	66	66	70	59	57.0	65.2	61.1
		4.5	75	71	69	39	77	74	74	33	63.5	64.5	64.0
		5	82	73	78	70	90	89	82	80	75.7	85.2	80.5
		Averages	64.3	64.8	62.0	58.7	69.9	62.8	68.8	60.2	62.4	65.4	63.9
	Per cent. of Error of Distance judged.							(205)					
		1	+13.0	+ 9.0	+14.0		+14.0	+21.0	+17.0		+12.0	+17.3	+14.6
		1.5	−18.3	−10.6	−15.3		− 1.3	− 3.3	−10.0		−14.7	− 4.9	− 9.8
		2	− 3.0	− 5.0	− 2.0		− 2.5	− .5	+ 2.0		− 3.3	− .3	− 1.8
		2.5	0	+ 2.0	+ 2.4	−1.2	+ 4.0	+ 3.3	0	+2.4	+ .8	+ 2.4	+ 1.6
		3	− 4.6	− 1.0	− 4.6	−1.3	− 3.6	− 8.3	− 5.3	−1.3	− 2.9	− 4.6	− 3.7
		3.5	+ 4.5	+ 1.5	+ 2.8	−1.7	+ 1.4	− 1.4	+ 7.4	+2.8	+ 1.8	+ 1.1	+ 1.4
		4	− 4.2	− 1.7	+ .2	− .8	+ .5	− .2	− 5	+2.2	− 1.6	+ .5	− .5
		4.5	− 2.7	+ 2.2	+ .2	+2.0	+ .4	+ 1.1	+ 7	+3.1	+ .4	+ 1.4	+ .9
		5	− 2.0	− 2.8	− 2.6	−3.2	− 1.0	− 1.3	− 2.2	−2.2	− 2.6	− 1.7	− 2.1
		Averages	− 1.0	− .7	− .5	−1.0	+ 1.3	+ 1.2	+ 1.0	+1.2	− 1.0	+ 1.2	+ .1
JUDGMENTS OF FIGURE.	No. times judged correctly per 100 times applied.							(208)					
		1	86	74	72		70	59	70		77.3	66.3	71.8
		1.5	85	70	74		98	97	98		76.3	97.7	87.0
		2	90	98	96		97	99	99		94.7	98.3	96.5
		2.5	100	100	100	98	100	100	100	100	99.5	100	99.8
		3	100	100	100	100	100	100	100	100	100	100	100
		3.5	100	100	100	100	100	100	100	100	100	100	100
		4	100	100	100	100	100	100	100	100	100	100	100
		4.5	100	100	100	100	100	100	100	100	100	100	100
		5	100	100	100	100	100	100	100	100	100	100	100
		Averages	95.7	93.6	93.6	99.7	96.1	95.0	96.3	100	95.6	96.8	96.2

Table 15. Experiment B.—Pins set in triangles and squares.

FOREARM (a) and (b).

(a) Pressing evenly three times only.

Person.	Distance (Centimeters).	Averages of N.										
Figure.		Triangles.				Squares.				T.	S.	T. and S.
No. Pins.		III	IV	VI	VII	IV	V	VIII	IX	Averages.		Total Averages.
Judgments of Number of Pins. No. times judged correctly per 100 times applied.						(209)						
	1	33	41	33		17	9	17				
	1.5	36	41	25		35	17	28				
	2	17	28	21		37	25	29				
	2.5	33	29	35	6	48	9	17	32			
	3	40	39	40	10	45	42	39	14			
	Averages	31.8	35.6	30.8	8.0	36.4	20.4	26.0	23.0	26.5	26.5	26.5
Per cent. of Error of No. Pins judged.						(210)						
	1	+37.3	+16.0	−24.0		+10.0	−17.6	−39.0				
	1.5	+44.0	+ 9.0	−22.7		+19.0	− 3.2	−32.0				
	2	+44.0	+19.0	− 6.7		+15.0	− 6.4	−34.0				
	2.5	+53.3	+25.0	−14.0	− 1.6	+ 9.0	− 4.8	−35.5	−48.0			
	3	+23.2	+ 1.0	−37.1	−11.2	+10.0	− 3.1	−33.3	−47.3			
	Averages	+40.4	+14.0	−20.9	− 6.4	+12.6	− 7.0	−34.7	−47.6	+ 6.8	−19.2	−1.2

JUDGMENTS OF DISTANCE.	No. times judged correctly per 100 times applied.	(211)											
		1	68	48	33		56	21	33		49.7	36.7	43.2
		1.5	21	33	13		37	31	39		22.3	35.6	29.0
		2	43	29	39		56	35	48		37.0	46.3	41.7
		2.5	52	29	25	29	51	45	45	33	33.7	43.5	38.6
		3	56	41	33	40	59	39	40	37	44.0	43.7	43.9
		Averages	48.0	36.0	29.8	34.5	51.8	34.2	41.0	35.0	37.1	40.5	38.8
	Per cent. of Error of Distance judged.	(212)											
		1	+32.0	+44.0	+68.0		+38.0	+60.0	+66.0		+48.0	+54.7	+51.3
		1.5	+ 8.0	+ 5.3	−10.7		+ 2.7	+10.7	+12.0		− .9	+ 8.5	+ 3.8
		2	− 6.0	− 5 0	− 1.8		−11.0	−10.0	− 5.0		− 4.3	− 8.7	− 6.5
		2.5	−12.8	−14.4	−23.2	−25.6	−11.2	−15.2	−14.4	−17.6	−19.0	−14.6	−16.8
		3	−13.0	−18.1	−30.1	−31.3	−20.1	−18.1	−19.5	−14.3	−23.1	−18.0	−20.5
		Averages	+ 1.6	+ 2.4	+ .4	−28.4	− .3	+ 5.5	+ 7.8	−16.0	+ .2	+ 4.5	+ 2.3
JUDGMENTS OF FIGURE.	No. times judged correctly per 100 times applied.	(213)											
		1	91	63	75		33	33	51		76.3	39.0	57.7
		1.5	64	67	53		36	33	52		61.3	40.3	51.0
		2	57	71	41		49	47	41		56.3	44.2	51.0
		2.5	65	49	67	56	45	44	53	55	59.2	49.2	54.2
		3	70	51	69	60	51	48	57	57	62.5	53.2	57.9
		Averages	69.4	60.2	61.0	58.0	42.8	41.0	50.8	56.0	62.1	47.6	54.8

Table 16. Experiment B. — Pins set in triangles and squares.

FOREARM.

(b) Attention given only to number of points felt.

Person.			Averages of N.										
Figure.		Distance (Centimeters).	Triangles.				Squares.				T.	S.	T. and S.
No. Pins.			III	IV	VI	VII	IV	V	VIII	IX	Averages.		Total Averages.
Judgments of Number of Pins.			(214)										
	No. times judged correctly per 100 times applied.	1	0	0	0		8	0	0				
		1.5	4	0	0		0	0	0				
		2	6	0	0		4	0	0				
		2.5	16	8	0	0	28	0	0	0			
		3	20	12	0	0	30	10	0	0			
		Averages	9.2	4.0	0	0	8.6	2.0	0	0	3.3	2.6	3.0
			(215)										
	Per cent. of Error of No. Pins judged.	1	+56.0	+44.0	+44.0		+76.0	+68.0	+72.0				
		1.5	+ 6.7	+ 4.0	+30.6		+25.3	+36.0	+28.0				
		2	−14.0	− 6.0	− 8.0		− 2.0	+ 6.0	+12.0				
		2.5	− 8.8	−10.4	−15.2	−26.0	+10.4	+ .8	− .8	+4.0			
		3	−12.0	−17.1	−21.0	−30.0	−12.4	−10.3	−21.0	−3.7			
		Averages	+ 5.6	+ 2.9	+ 6.1	−28.0	+19.5	+22.2	+18.0	+ .2	−.8	+15.0	+7.1

Table 17a. Experiment B.—Pins set in triangles and squares.

SUMMARY.

Averages brought forward from foregoing tables, 11 to 16, inclusive.

REGION.	GENERAL AVERAGES FROM WHOLE SET OF DISTANCES WORKED ON (1 TO 5 CMTRS.), AND FROM THE FOUR PERSONS (N. P. L. AND B.).										
FIGURE.	TRIANGLES.				SQUARES.				T.	S.	T. AND S.
NO. PINS.	III	IV	VI	VII	IV	V	VIII	IX	Averages.		Total Averages.
JUDGMENTS OF NUMBER OF PINS. No. times judged correctly per 100 times applied.						(216)					
Tongue	99.2	99.4	98.2	97.5	99.4	99.2	98.6	98.0	98.6	98.8	98.7
Forehead	59.3	50.7	46.7	49.7	62.5	68.0	78.5	71.0	51.6	70.0	60.8
Forearm	40.0	42.5	43.8	25.3	49.2	49.2	46.5	39.3	37.9	46.0	42.0
Abdomen	43.4	43.0	41.2	20.0	50.1	40.3	53.0	40.7	36.9	46.0	41.5
Forearm (*a*)	31.8	35.6	30.8	8.0	36.4	20.4	26.0	23.0	26.5	26.5	26.5
Forearm (*b*)	20.0	12.0	0	0	30.0	10.0	0	0	8.0	11.0	9.0
Averages	48.95	47.20	43.45	33.41	54.60	47.85	50.43	45.33	43.25	49.55	46.41
Per cent. of Error of Distance judged.						(217)					
Tongue	+ .2	+ .1	− .2	− .3	+ .1	+ .1	− .1	− .2	− .5	− .25	− .375
Forehead	+27.6	+11.1	+ 5.2	− 9.6	+20.2	+ 9.3	− 4.5	− 1.0	+5.6	+ 6.0	+5.9
Forearm	+31.7	+10.8	−10.6	− 9.6	+23.9	+ 8.9	−11.5	−13.5	+5.6	+ 1.3	+3.45
Abdomen	+33.9	+ 8.7	−14.8	−22.5	+26.0	+ 9.1	−10.1	−10.7	+1.3	+ 3.3	+ .23
Forearm (*a*)	+40.4	+14.0	−20.9	− 6.4	+12.6	− 7.0	−34.7	−47.6	+6.8	−19.2	−6.20
Forearm (*b*)	+ 5.6	+ 2.9	+ 6.1	−28.0	+19.5	+22.2	+18.0	+ .2	− .8	+15.0	+7.1
Averages	+23.2	+ 7.9	− 5.9	−12.7	+17.0	+ 7.1	− 7.1	−12.1	+3.1	+ 1.1	+2.1

JUDGMENTS OF DISTANCE.

No. times judged correctly per 100 times applied.

(218)											
Tongue	98.6	98.4	97.8	97.5	98.8	98.2	97.8	99.0	98.1	98.9	98.5
Forehead	81.0	76.2	77.2	76.7	80.7	77.7	79.3	80.7	77.8	79.6	78.7
Forearm	75.3	75.5	77.7	72.3	78.3	76.3	79.3	76.0	75.2	77.5	76.3
Abdomen	64.3	64.8	62.0	58.7	69.9	62.8	68.8	60.2	62.4	65.4	63.9
Forearm (*a*)	48.0	36.0	29.8	34.5	51.8	34.2	41.0	35.0	37.1	40.5	38.8
Forearm (*b*)											
Averages	73.4	70.18	68.9	67.94	75.90	69.84	73.24	70.18	70.12	72.38	71.2

Per cent. of Error of Distance judged.

(219)											
Tongue	+ .8	+ .5	− .9	− 1.0	+ .7	+ .5	−1.1	− .3	− .1	0	− .05
Forehead	+1.5	+ .1	−2.7	− 1.4	+ .7	+ .5	+ .2	+ .2	− .6	+ .4	− .1
Forearm	+2.6	+2.1	+1.5	+ .3	+4.2	+3.9	+3.2	− .2	+1.6	+2.8	+2.2
Abdomen	−1.9	− .7	− .5	− 1.0	+1.3	+1.2	+1.0	+ 1.2	−1.0	+1.2	+ .1
Forearm (*a*)	+1.6	+2.4	+ .4	−28.4	− .3	+5.5	+7.8	−16.0	−6.0	− .7	−3.35
Forearm (*b*)											
Averages	+ .9	+ .9	− .4	− 7.9	+1.3	+2.3	+2.2	− 3.0	−1.2	+ .7	− .2

JUDGMENTS OF FIGURE.

No. times judged correctly per 100 times applied.

(220)											
Tongue	100	100	100	100	100	100	100	100	100	100	100
Forehead	97.8	95.5	95.2	99.7	98.8	98.3	97.3	100	97.0	98.6	97.8
Forearm	94.0	93.5	90.3	98.3	95.0	93.3	95.2	99.0	94.0	95.6	94.8
Abdomen	95.7	93.6	93.6	99.7	96.1	95.0	96.3	100	95.6	96.8	96.2
Forearm (*a*)	69.4	60.2	61.0	58.0	42.8	41.0	50.8	56.0	62.1	47.6	54.8
Forearm (*b*)											
Averages	91.4	88.6	88.0	91.14	86.5	85.5	87.9	91.0	89.7	87.7	88.7

Table 17b. Experiment B. — Pins set in triangles and squares.

SUMMARY.

Averages brought forward from foregoing tables, 11 to 16 inclusive.

Region.		General Averages from the Various Numbers of Pins Worked with, and from the Four Persons (N. P. L. and B.).											
		Judgments of Distance.									T.	S.	T. and S.
Distances (cmtrs.).		1	1.5	2	2.5	3	3.5	4	4.5	5	Averages		Total Averages
Judgments of Distance. No. times judged correctly per 100 times applied.							(221)						
	Tongue	98.2	97.0	97.2	99.0	99.4					98.1	98.9	98.5
	Forehead	86.2	79.2	71.8	68.4	73.0	93.8				77.8	79.6	78.7
	Forearm	69.6	74.3	74.8	69.2	74.8	94.5				75.2	77.5	76.3
	Abdomen	68.7	61.5	69.8	66.8	56.5	51.9	61.1	64.0	80.5	62.4	65.4	63.9
	Forearm (*a*)	43.2	29.0	41.7	38.6	43.9					37.1	40.5	38.8
	Forearm (*b*)												
	Averages	73.1	68.2	71.1	68.4	69.5	80.1	61.1	64.0	80.5	70.12	72.38	71.2
Per cent. of Error of Distance judged.							(222)						
	Tongue	+ .25	0	+ .2	− .1	− .4					0	0	0
	Forehead	+ 4.7	−1.2	−2.5	+ .25	− .4	− .85				− .6	+ .4	− .1
	Forearm	+15.5	+ .95	+1.5	+ 1.8	− .1	− .7				+1.6	+2.8	+2.2
	Abdomen	+14.65	−9.8	−1.8	+ 1.6	− 3.75	+1.45	−.5	+.9	−2.1	−1.0	+1.2	+ .1
	Forearm (*a*)	+51.35	+3.8	−6.5	−16.8	−20.55					+ .2	+4.5	+2.3
	Forearm (*b*)												
	Averages	+17.3	−1.2	−1.8	−2.6	− 5.0	0	−.5	+.9	−2.1	0	+1.8	+ .9

(223)

JUDGMENTS OF FIGURE. No. times judged correctly per 100 times applied.												
Tongue	100	100	100	100	100					100	100	100
Forehead	89.2	96.7	98.3	99.9	99.8	99.9				97.0	98.6	97.8
Forearm	82.0	91.2	95.5	97.9	99.2	97.5				94.0	95.6	94.8
Abdomen	71.8	87.0	96.5	99.8	100	100	100	100	100	95.6	96.8	96.2
Forearm (*a*)	57.7	51.2	51.0	54.2	57.9					62.1	47.6	54.8
Forearm (*b*)												
Averages	80.1	85.2	88.2	90.3	93.3	99.1	100	100	100	89.74	87.72	88.73

EXPERIMENT C.

WITH LINEAL FIGURES.

Apparatus. — The lineal figures were made from cardboard of medium thickness, but very hard and strong. This material was chosen to avoid temperature complications. Great care was taken that the lines, and particularly the corners of the figures, should be perfectly even, sharp, and accurate throughout. The importance of this cannot be fully appreciated unless one has acted as subject for a long period. It is little less than marvelous how slight a cue will be noted by which to remember a particular piece of apparatus as "that same old one," and so the judgments become based upon a fund of past experiences and imaginations, rather than upon a new and present impression, as is absolutely necessary for the work here in hand. In our work, if any piece became thus "individualized," it was at once discarded. The figures were made like deep pasteboard boxes, with one end (that to be pressed on the skin) left open, as when the lid of the box is off. The larger pieces were braced, as it were, with false bottoms, one or more, as needed to make them firm.

The figures used were triangles, squares, and circles. The categories of "distance" remained the same as in the previous experiments. There were no longer, of course, any "number" categories to be observed.

Method. — This was precisely the same as in Experiment B, but a new care was required in applying the apparatus to the subject. The pieces being made hollow, like a drum, they would, upon the least slipping of the fingers over their surface while handling them, give out a sound with the spontaneity of a resonance box or a tambourine. This sound would become individualized by the subject for each particular piece, the same as a bent corner or an imperfect line, and, in a way making the judgments worthless if such sounds were permitted. The utmost care, therefore, was used throughout, in handling the pieces, to avoid every particle of slipping or rubbing of the box, either upon the subject's skin, or upon the fingers of the operator.

The tongue was no longer investigated, as the apparatus now was too large to work with comfortably in the mouth.

The Tables 18a to 22a, inclusive, for Experiment C, and 18b to 22b for Experiment D, will be understood, after examination of the similar ones for Experiments A and B, without further explanation. These above-numbered tables of Experiments C and D correspond, respectively, to Tables 2, 3, 4, 5, and 10 of Experiment

A, and to Tables 12, 13, 14, 15, and 17 of Experiment B.

Table 23. This table shows the distribution of the whole number of figure-judgments. They are calculated, in per cent., from one hundred applications of each piece of apparatus to each of the several regions of skin worked upon. For example: the three numbers, 65.0, 21.0, 14.0, arranged vertically in the upper left-hand corner of the table, mean that of the total number of times that the "1 centimeter" triangle was applied to the various regions of the body, in 65 per cent. of those times, this triangle was judged correctly to be a triangle; in 21 per cent. it was misjudged to be a square; and in 14 per cent. a circle.

The purpose of this table (to be discussed in our general study) is to aid in comprehending the errors made in judging the figures.

The heavy figures in this table show the correct judgments; the other figures show the false judgments.

EXPERIMENT D.

WITH SOLID FIGURES.

Apparatus. — Like that for Experiment A, except that the pieces were made of cork, and solid throughout.

Method. — Precisely that of Experiment C.

Tables. — See "*The Tables*" (page 41) under Experiment C.

Table 18a. Experiment C.—With lineal figures.

FOREHEAD.

Person / Figure		Distance (Centimeters)	Averages of N. and P. — Triangles	Squares	Circles	Averages of T., S. and C.
Judgments of Distance.	No. times judged correctly per 100 times applied.			(226)		
		1	86	32	72	63.3
		1.5	52	50	54	52.0
		2	56	32	64	50.7
		2.5	50	30	38	39.3
		3	42	44	48	44.7
		3.5	48	88	62	66.0
		Averages	55.7	46.0	56.3	52.7
	Per cent. of Error of Distance judged.			(229)		
		1	+ 7.0	+ 42.0	+ 16.0	+ 21.7
		1.5	− 4.0	+ 20.0	+ 12.6	+ 9.5
		2	+ .5	+ 28.5	+ 5.0	+ 11.3
		2.5	− 1.6	+ 17.6	+ 4.8	+ 6.9
		3	− 1.0	+ 15.3	− .6	+ 4.6
		3.5	− 8.2	− 1.7	− 5.7	− 5.2
		Averages	− 1.3	+ 20.3	+ 5.2	+ 8.1
Judgments of Figure.	No. of correct judgments per 100 times applied.			(232)		
		1	74	32	54	
		1.5	70	46	38	
		2	68	64	54	
		2.5	60	72	86	
		3	72	78	80	
		3.5	54	82	84	
		Averages	66.3	62.3	66.0	64.87

Table 18b. Experiment D. — With solid figures.

FOREHEAD.

Person / Figure		Distance (Centimeters)	Averages of N. and P.: Triangles	Squares	Circles	Averages of T., S. and C.
Judgments of Distance.	No. times judged correctly per 100 times applied.			(235)		
		1	52	34	56	47.3
		1.5	52	30	44	42.0
		2	22	26	40	29.3
		2.5	22	28	14	21.3
		3	34	28	42	34.7
		3.5	84	80	78	80.7
		Averages	44.3	37.7	45.7	42.6
	Per cent. of Error of Distance judged.			(238)		
		1	+32.0	+53.0	+29.0	+38.0
		1.5	+ 2.0	+38.5	+25.3	+21.9
		2	+22.0	+40.0	−17.0	+26.3
		2.5	+11.6	+21.2	+12.8	+15.2
		3	+ 6.3	+ 8.3	+ 3.0	+ 5.9
		3.5	− 2.8	− 3.4	− 3.1	− 3.1
		Averages	+11.8	+26.3	+14.0	+17.4
Judgments of Figure.	No. of correct judgments per 100 times applied.			(241)		
		1	74	40	54	
		1.5	68	58	66	
		2	62	68	84	
		2.5	76	82	82	
		3	82	92	86	
		3.5	88	94	94	
		Averages	75.0	72.3	77.7	75.0

Table 19a. Experiment C. — With lineal figures.

FOREARM.

Person		Distance	Averages of N. and P.			
Figure		(Centimeters)	Triangles	Squares	Circles	Averages of T., S. and C.
Judgments of Distance.	No. times judged correctly per 100 times applied.			(244)		
		1	44	32	30	35.3
		1.5	36	28	30	31.3
		2	40	38	42	40.0
		2.5	38	26	32	32.0
		3	20	38	48	35.3
		3.5	12	76	28	38.7
		Averages	31.6	39.6	35.0	35.4
	Per cent. of Error of Distance judged.			(247)		
		1	+ 40.0	+ 54.0	+ 71.0	+ 55.0
		1.5	+ 24.6	+ 34.0	+ 12.7	+ 23.8
		2	+ 11.0	+ 22.5	+ 10.0	+ 14.5
		2.5	− 1.6	+ 9.2	− 3.2	+ 1.5
		3	− 18.7	+ 3.7	− 7.3	− 7.4
		3.5	− 23.4	− 4.9	− 18.0	− 15.4
		Averages	+ 5.3	+ 19.7	+ 10.9	+ 12.0
Judgments of Figure.	No. of correct judgments per 100 times applied.			(250)		
		1	62	56	40	
		1.5	58	48	44	
		2	68	48	58	
		2.5	44	54	48	
		3	38	56	60	
		3.5	26	76	82	
		Averages	49.3	56.3	55.3	53.6

Table 19b. Experiment D. — With solid figures.

FOREARM.

Person / Figure		Distances (Centimeters)	Averages of N. and P. — Triangles	Squares	Circles	Averages of T., S. and C.
Judgments of Distance.	No. times judged correctly per 100 times applied.			(253)		
		1	38	30	40	36.0
		1.5	32	36	26	31.3
		2	38	32	30	33.3
		2.5	14	24	32	23.3
		3	58	50	54	54.0
		3.5	38	80	80	66.0
		Averages	36.3	42.0	43.7	40.7
	Per cent. of Error of Distance judged.			(256)		
		1	+ 53.0	+ 65.0	+ 52.0	+ 56.7
		1.5	+ 9.3	+ 36.7	+ 26.6	+ 24.2
		2	+ 14.0	+ 22.5	+ 4.0	+ 13.5
		2.5	− 9.2	+ 26.8	+ 8.8	+ 8.8
		3	− 6.0	+ 6.3	− 2.0	− .6
		3.5	− 13.4	− 4.0	− 3.4	− 6.9
		Averages	+ 7.9	+25.5	+14.3	+15.9
Judgments of Figure.	No. of correct judgments per 100 times applied.			(259)		
		1	52	36	42	
		1.5	56	44	56	
		2	44	38	66	
		2.5	36	38	62	
		3	64	68	80	
		3.5	54	68	88	
		Averages	51.0	48.7	65.7	55.1

Table 20a. Experiment C. — With lineal figures.

ABDOMEN.

Person		Distances	Averages of N. and P.			
Figure		(Centimeters)	Triangles	Squares	Circles	Averages of T., S. and C.
Judgments of Distance.	No. times judged correctly per 100 times applied.			(262)		
		1	50	48	42	46.7
		1.5	36	32	46	38.0
		2	36	26	32	31.3
		2.5	30	40	42	37.3
		3	30	40	40	36.7
		3.5	36	76	34	48.7
		Averages	36.3	43.7	39.3	39.8
	Per cent. of Error of Distance judged.			(265)		
		1	+ 43.0	+ 44.0	+ 52.0	+ 46.3
		1.5	+ 19.3	+ 17.3	+ 18.6	+ 18.4
		2	− 1.5	+ 28.5	+ 1.5	+ 9.5
		2.5	− 3.6	+ 13.2	+ .8	+ 3.5
		3	− 10.0	+ 6.6	− 3.0	− 2.1
		3.5	− 17.7	− 5.4	− 13.1	− 12.1
		Averages	+ 4.9	+ 17.4	+ 9.4	+ 10.6
Judgments of Figure.	No. times judged correctly per 100 times applied.			(268)		
		1	52	38	36	
		1.5	72	52	64	
		2	48	46	86	
		2.5	74	60	82	
		3	56	72	92	
		3.5	46	78	80	
		Averages	58.0	57.7	73.3	63.0

Table 20b. Experiment D. — With solid figures.

ABDOMEN.

Person / Figure		Distance (Centimeters)	Averages of N. and P. — Triangles	Squares	Circles	Averages of T., S. and C.
Judgments of Distance.	No. times judged correctly per 100 times applied.			(271)		
		1	34	44	46	41.3
		1.5	24	28	32	28.0
		2	54	30	52	45.3
		2.5	26	32	34	30.7
		3	32	38	48	39.3
		3.5	48	74	68	63.3
		Averages	36.3	41.0	46.7	41.3
	Per cent. of Error of Distance judged.			(274)		
		1	+55.0	+48.0	+36.0	+46.3
		1.5	+ .6	+26.0	+26.6	+17.7
		2	+ 2.5	+27.0	− 1.5	+ 9.3
		2.5	−12.0	+11.2	− 2.0	− .9
		3	− 5.3	+ 6.3	−10.0	− 3.0
		3.5	−15.7	− 4.9	− 6.2	− 8.9
		Averages	+ 4.2	+19.0	+ 7.1	+10.1
Judgments of Figure.	No. times judged correctly per 100 times applied.			(277)		
		1	58	34	46	
		1.5	62	42	42	
		2	50	46	62	
		2.5	68	62	70	
		3	66	66	72	
		3.5	52	72	90	
		Averages	59.3	53.7	63.7	58.9

Table 21a. Experiment C. — With lineal figures.

(a) FOREARM.

Pressing evenly three times only.

Person		Distance	Averages of N. and P.			
Figure		(Centimeters)	Triangles	Squares	Circles	Averages of T., S. and C.
Judgments of Distance.	No. times judged correctly per 100 times applied.			(280)		
		1	54	30	30	38.
		1.5	30	26	26	27.3
		2	48	36	50	44.7
		2.5	24	14	16	18.0
		3	24	52	44	40.
		3.5	22	54	22	32.7
		Averages	33.7	35.3	31.3	33.4
	Per cent. of Error of Distance judged.			(283)		
		1	+ 41.0	+ 62.0	+ 68.0	+ 57.0
		1.5	+ 12.7	+ 27.3	+ 22.7	+ 20.9
		2	+ 3.5	+ 21.0	+ 7.5	+ 10.7
		2.5	− 7.6	+ 10.8	+ 4.8	+ 2.7
		3	− 13.6	− 2.0	− 6.6	− 7.3
		3.5	− 21.4	− 3.7	− 18.0	− 14.4
		Averages	+ 2.4	+ 19.2	+ 13.1	+ 11.6
Judgments of Figure.	No. of correct judgments per 100 times applied.			(286)		
		1	72	36	48	
		1.5	60	38	50	
		2	54	56	52	
		2.5	52	50	56	
		3	50	60	52	
		3.5	39	60	66	
		Averages	53.0	50.0	54.0	52.3

Table 21b. Experiment D. — With solid figures.

(a) FOREARM.

Pressing evenly three times only.

Person / Figure		Distances (Centimeters)	Averages of N. and P.: Triangles	Squares	Circles	Averages of T., S. and C.
Judgments of Distance.	No. times judged correctly per 100 times applied.			(289)		
		1	54	44	22	40.0
		1.5	23	22	24	23.0
		2	30	18	32	26.7
		2.5	48	26	30	34.7
		3	32	52	50	44.7
		3.5	20	80	46	48.7
		Averages	34.5	40.3	34.0	36.3
	Per cent. of Error of Distance judged.			(292)		
		1	+ 46.0	+ 49.0	+ 64.0	+ 53.0
		1.5	+ 9.3	+ 40.6	+ 31.3	+ 27.1
		2	+ 8.0	+ 33.0	+ 21.5	+ 20.8
		2.5	− 1.6	+ 14.4	+ 7.2	+ 6.7
		3	− 11.0	+ 8.3	− 5.3	− 2.7
		3.5	− 16.3	− 4.0	− 3.4	− 7.9
		Averages	+ 5.7	+ 23.5	+ 19.2	+ 16.1
Judgments of Figure.	No. of correct judgments per 100 times applied.			(295)		
		1	46	44	40	
		1.5	42	36	32	
		2	46	28	44	
		2.5	40	36	52	
		3	48	56	36	
		3.5	24	66	60	
		Averages	41.0	44.3	44.0	43.1

Table 22a. Experiments C and D.—With lineal and solid figures.

SUMMARY.—Averages brought forward from Tables 18a to 21b.

		Person. / Figure.		Average of N. and P. Tri-angles	Squares	Circles	Averages of T. S. and C.
Judgments of Distance.	No. times judged correctly per 100 times applied.					(298)	
		Forehead	Lineal	55.7	46.0	56.3	52.7
			Solid	44.3	37.7	45.7	42.6
		Forearm	Lineal	31.6	39.6	35.0	35.4
			Solid	36.3	42.0	43.7	40.7
		Forearm (*a*)	Lineal	33.7	35.3	31.3	33.4
			Solid	34.5	40.3	34.0	36.3
		Abdomen	Lineal	36.3	43.7	39.3	39.8
			Solid	36.3	41.0	46.7	41.3
		Averages	Lineal	39.325	41.15	40.47	40.30
			Solid	37.850	40.25	42.52	40.26
		Average of Lineal and Solid		38.590	40.70	41.50	40.28
	Per cent. of Error of Distance Judgments.					(301)	
		Forehead	Lineal	— 1.3	+20.3	+ 5.2	+ 8.1
			Solid	+11.8	+26.3	+14.0	+17.4
		Forearm	Lineal	+ 5.3	+19.7	+10.9	+12.0
			Solid	+ 7.9	+25.5	+14.3	+15.9
		Forearm (*a*)	Lineal	+ 2.4	+19.2	+13.1	+11.6
			Solid	+ 5.7	+23.5	+19.2	+16.1
		Abdomen	Lineal	+ 4.9	+17.4	+ 9.4	+10.6
			Solid	+ 4.2	+19.0	+ 7.1	+10.1
		Averages	Lineal	+ 2.8	+19.1	+ 9.6	+10.5
			Solid	+ 7.4	+23.6	+13.6	+14.9
		Average of Lineal and Solid		+ 5.1	+21.3	+11.6	+12.7
Judgments of Figure.	No. of correct judgments per 100 times applied.					(304)	
		Forehead	Lineal	66.3	62.3	66.0	64.9
			Solid	75.0	72.3	77.7	75.0
		Forearm	Lineal	49.3	56.3	55.3	53.6
			Solid	51.0	48.7	65.7	53.1
		Forearm (*a*)	Lineal	53.0	50.0	54.0	52.3
			Solid	41.0	44.3	44.0	43.1
		Abdomen	Lineal	58.0	57.7	73.3	63.0
			Solid	59.3	53.7	63.7	58.9
		Averages	Lineal	56.65	56.57	62.15	58.45
			Solid	56.57	54.75	62.77	58.025
		Average of Lineal and Solid		56.61	55.66	62.46	58.20

Table 22 b. Experiments C and D.—With lineal and solid figures.

SUMMARY. — Averages brought forward from Tables 18a to 21b.

Judgments of Distance.				Distances (Centimeters). 1	1.5	2	2.5	3	3.5	Averages.
No. times judged correctly per 100 times applied.							(305)			
	Forehead	Lineal		63.3	52.0	50.7	39.3	44.7	66.0	52.7
		Solid		47.3	42.0	29.3	21.3	34.7	80.7	42.6
	Forearm	Lineal		35.3	31.3	40.0	32.0	35.3	38.7	35.4
		Solid		36.0	31.3	33.3	23.3	54.0	66.0	40.7
	Forearm (a)	Lineal		38.0	27.3	44.7	18.0	40.0	32.7	33.4
		Solid		40.0	23.0	26.7	34.7	44.7	48.7	36.3
	Abdomen	Lineal		46.7	38.0	31.3	37.3	36.7	48.7	39.8
		Solid		41.3	28.0	45.3	30.7	39.3	63.3	41.3
	Averages	Lineal		45.8	37.1	41.7	31.7	39.2	46.5	40.30
		Solid		41.2	31.1	33.7	27.5	43.2	64.7	40.26
	Average of Lineal and Solid			43.5	34.1	37.7	29.6	41.2	55.6	40.28
Per cent. of Error of Distance Judgments.							(306)			
	Forehead	Lineal		+21.7	+ 9.5	+11.3	+ 6.9	+4.6	− 5.2	+ 8.1
		Solid		+38.0	+21.9	+26.3	+15.2	+5.9	− 3.1	+17.4
	Forearm	Lineal		+55.0	+23.8	+14.5	+ 1.5	−7.4	−15.4	+12.0
		Solid		+56.7	+24.2	+13.5	+ 8.8	− .6	− 6.9	+15.9
	Forearm (a)	Lineal		+57.0	+20.9	+10.7	+ 2.7	−7.3	−14.4	+11.6
		Solid		+53.0	+27.1	+20.8	+ 6.7	−2.7	− 7.9	+16.1
	Abdomen	Lineal		+46.3	+18.4	+ 9.5	+ 3.5	−2.1	−12.1	+10.6
		Solid		+46.3	+17.7	+ 9.3	− .9	−3.0	− 8.9	+10.1
	Averages	Lineal		+45.0	+18.1	+11.5	+ 3.6	−3.0	−11.8	+10.5
		Solid		+48.5	+22.7	+17.5	+ 7.4	− .1	− 6.7	+14.9
	Average of Lineal and Solid			+46.75	+20.4	+14.5	+ 5.5	−1.6	− 9.2	+12.7

Table 23. Experiments C and D—

(For explanation of this

Distances.		1			1.5			2		
Figures.		T.	S.	C.	T.	S.	C.	T.	S.	C.
			(307)			(308)			(309)	
Exp. C.	Triangle	**65.0**	41.0	24.5	**65.0**	36.0	26.0	**59.5**	32.0	13.5
	Square	21.0	**40.5**	31.0	28.0	**46.0**	25.0	32.5	**53.5**	24.0
	Circle	14.0	18.5	**44.5**	7.0	18.0	**49.0**	8.0	14.5	**62.5**
			(314)			(315)			(316)	
Exp. D.	Triangle	**57.5**	45.5	24.5	**57.0**	35.0	25.5	**50.5**	33.5	20.0
	Square	27.0	**38.5**	30.0	27.5	**45.0**	25.5	32.5	**45.0**	16.0
	Circle	15.5	16.0	**45.5**	15.5	20.0	**49.0**	17.0	21.5	**64.0**

With lineal and solid figures.

table, see page 42.)

2.5			3			3.5			Averages of all Distances.			Total Aver-ages.
T.	S.	C.	T.	S.	C.	T.	S.	C.	T.	S.	C.	
(310)			(311)			(312)			(313)			
57.5	21.5	4.0	**54.0**	16.0	11.5	**39.0**	12.5	5.0	**56.6**	26.5	14.1	58.5
32.5	**59.0**	28.0	38.0	**66.5**	17.5	44.0	**74.0**	17.0	32.6	**56.6**	23.7	
10.0	19.5	**68.0**	8.0	17.5	**71.0**	17.0	13.5	**58.0**	10.6	16.9	**62.1**	
(317)			(318)			(319)			(320)			
55.0	30.5	14.5	**65.0**	13.0	10.5	**54.5**	19.0	5.0	**56.6**	29.4	16.6	57.5
28.5	**45.0**	19.0	31.0	**70.5**	21.0	38.0	**75.0**	12.0	30.7	**53.1**	20.6	
16.5	24.5	**66.5**	4.0	16.5	**68.5**	7.5	6.0	**83.0**	12.6	17.4	**62.7**	

EXPERIMENT E.

WITH A MOVING PENCIL.

At a certain stage in our studies, we shall have to inquire what part each of several elements, which we know enter into the formation of every judgment, individually plays. Important among these are "mass," both of stimulations and of feelings; "intensity," both peripheral and central; the "time rate" of stimulation, and of mental response. Particularly we shall wish to know the separate influence of each of these factors, in order to comprehend their united action in producing a judgment as a whole. Experiment E, still pursuing the comparative method of investigation, has this exigency in view. It differs from all the foregoing experiments by introducing motion over the skin.

The apparatus and the method were of the simplest kinds. The pencil was of ivory, 5 millimeters in diameter, and rounded hemispherically at the end. It was always kept at the skin temperature, was held vertical, and applied by an assistant. The region to be worked upon was laid out in squares by dots of ink one centimeter apart. The pencil was always drawn in the same direction, *i.e.*, horizontally on the forehead, and down

on the forearm and the abdomen. Four categories of motion were investigated: Quick and Heavy; Quick and Light; Slow and Heavy; Slow and Light. The "quick" and "slow" movements were timed by a metronome, until we had, by continued habit, well acquired the beat. "Quick" was at the rate of about 20 centimeters per second; and slow about 2 centimeters per second. No attempt was made to gauge the pressure exactly. "Heavy" was as hard as could be borne for a length of time without pain. "Light" was as light as could be distinctly and evenly felt.

The tables are so like the other tables that little further explanation will be needed. As only two subjects were available at the time the experiment was performed, a double number of applications was made, and each person and their results kept separate, as is shown in the tables.

Table 24. Experiment E. — With a moving pencil.

FOREHEAD.

Person. / Mode of Motion.		Distance (Centimeters)	Average of N. and P.: Quick and Light	Quick and Heavy	Slow and Light	Slow and Heavy	Averages
Judgments of Distance.	No. times judged correctly per 100 times applied.				(323)		
		1	96	96	94	92	94.5
		2	82	86	80	80	82.0
		3	74	82	80	76	78.0
		4	56	64	72	62	63.5
		5	68	78	76	56	69.5
		6	90	94	90	94	92.5
		Averages	77.67	83.33	82.0	76.67	80.0
	Per cent. of Error of Distance judged.				(326)		
		1	+ 4.0	+ 4.0	+ 6.0	+ 8.0	+ 5.5
		2	+ 7.0	+ 1.0	+ 4.0	+ 8.0	+ 1.5
		3	− 4.6	0	+ 1.3	+ 7.3	+ 1.0
		4	− 6.5	+ 6.0	− 1.0	+ 6.0	+ 1.1
		5	− 1.6	+ 1.2	+ 2.4	+ 6.4	+ 2.1
		6	− 2.0	− 1.0	− 1.6	− 1.0	− 1.4
		Averages	− 2.9	+ 1.9	+ 1.8	+ 5.8	+ 1.6

Table 25. Experiment E. — With a moving pencil.

FOREARM.

Person. / Mode of Motion.		Distance (Centimeters)	Average of N. and P. Quick and Light	Quick and Heavy	Slow and Light	Slow and Heavy	Averages
Judgments of Distance.	No. times judged correctly per 100 times applied.				(331)		
		1	91	88	74	61	78.5
		2	55	77	53	52	59.25
		3	60	56	48	47	52.75
		4	38	46	39	35	39.5
		5	50	46	42	43	45.25
		6	42	86	29	63	55.0
		Averages	56.0	66.5	47.5	50.2	55.0
	Per cent. of Error of Distance judged.				(336)		
		1	+ 9.0	+13.0	+27.0	+47.0	+24.0
		2	− 8.5	+ 7.5	+ 7.0	+21.5	+ 6.9
		3	−10.0	+11.0	+ 2.6	+15.5	+ 4.8
		4	− 8.2	+ 5.5	− 3.5	+ 8.2	+ .5
		5	− 8.8	+ 2.2	− 9.2	+ 1.0	− 3.9
		6	−13.8	− 3.0	−16.2	− 9.1	−10.3
		Averages	− 6.7	+ 6.0	+ 1.3	+14.0	+ 3.7

Table 26. Experiment E. — With a moving pencil.

ABDOMEN.

Person.		Distance (Centimeters)	Average of N. and P.				
Mode of Motion.			Quick and Light	Quick and Heavy	Slow and Light	Slow and Heavy	Averages
Judgments of Distance.	No. times judged correctly per 100 times applied.				(341)		
		1	69	63	47	49	57.0
		2	66	72	49	51	59.5
		3	53	52	45	48	49.5
		4	32	43	39	39	38.25
		5	43	39	39	44	41.25
		6	44	62	39	61	51.5
		Averages	51.17	55.17	42.98	48.67	49.5
	Per cent. of Error of Distance judged.				(346)		
		1	+31.0	+39.0	+59.0	+58.0	+46.7
		2	+ 9.0	+10.5	+24.5	+26.0	+17.5
		3	+ 2.0	+ 7.3	+ 8.3	+15.0	+ 8.1
		4	− 8.2	+ 3.5	0	+ 5.5	+ .2
		5	−11.0	− 3.4	− 9.0	+ 2.2	− 5.3
		6	−15.8	− 8.3	−15.0	− 9.3	−12.1
		Averages	+ 1.2	+ 8.1	+11.3	+16.2	+ 9.2

Table 27. Experiment E.—With a moving pencil.

SUMMARY.

Averages brought forward from Tables 24, 25 and 26.

Person.			Averages of N. and P.				
Mode of Motion.			Quick and Light	Quick and Heavy	Slow and Light	Slow and Heavy	Averages
Judgments of Distance.	No. times judged correctly per 100 times applied.				(351)		
		Forehead	77.67	83.33	82.0	76.67	80.0
		Forearm	56.0	66.5	47.5	50.2	55.0
		Abdomen	51.17	55.17	42.98	48.67	49.5
		Averages	61.6	68.3	57.5	58.5	61.5
	Per cent. of Error of Distance judged.				(356)		
		Forehead	−2.9	+1.9	+ 1.8	+ 5.8	+1.6
		Forearm	−6.7	+6.0	+ 1.3	+14.0	+3.7
		Abdomen	+1.2	+8.1	+11.3	+16.2	+9.2
		Averages	−2.8	+5.3	+ 4.8	+12.0	+4.8

EXPERIMENT F.

COMPARING HORIZONTAL AND VERTICAL DISTANCE.

Apparatus.—Fifteen pieces of hard, thin cardboard, cut accurately into strips as follows: five exactly 1 centimeter wide, five 2, and five 3 centimeters wide. Then each piece was again cut so that one end of each of the five cards of the three distance-categories, should be respectively, 1, 2, 3, 4 and 5 millimeters shorter than the standard distance, to which the other end of the card was originally, and still remains cut. It will be seen that, by applying first one end and then the other of each of these sets of five cards in regular order, increasing ratios of difference between the two distances, thus successively presented for judgment, would obtain throughout each set; that is for each standard distance.

Method.—The standard and longer end of each card was always pressed on the skin vertically, that is "up and down" of the body or limb, wherever applied, and in whatever order the two ends of the card might be applied. The card was applied by the assistant in a manner like that described for other apparatus in the foregoing experiments. The subject always compared the *second* application with the *first*; announcing that

the second was "more" or "less" than the first, or that "no difference" could be felt.

As only relative results were sought for, and no absolute threshold distance was concerned, we neglected all the "no difference" judgments. Only the answers "more" and "less" were recorded or counted in our series of 100 applications.

Two categories of application were used. In the first of these, marked always $\frac{V}{H}$ in the left-hand column of Table 28, the order of applying each card was, first the *long* end vertically, and then the *short* end horizontally. In the second of these direction-categories, marked always $\frac{H}{V}$ in the said column, the order was first the *short* end of the card horizontally, then the *long* end vertically. As, therefore, in the first category all the answers ought to be "less," and in the second category all ought to be "more," the results of the two series ought to balance each other.

Table 28 shows the results of this Experiment F. The numbers expressed as fractions in the body of the table mean as follows: the numerator gives the number of times, out of the 100 applications of each card, that the second application was judged to be "more" than the first. The denominator gives the number of times the second application was judged to be "less" than the first. The numerator plus the denominator always equals 100.

Since in the first line of fractions (see $\frac{V}{H}$, Table 28) the longer end of the card was applied first, and then the shorter end compared with it, and in the second line ($\frac{H}{V}$ in tables) the order of applying the different ends of the cards was reversed, the shorter end being now compared with the longer, plainly the sum of each vertical pair of these two horizontal lines of fractions ought to reduce to unity, and the sum of the whole two lines also reduce to unity, — that is, if there were no errors of judgment. If every judgment was correct, all the fractions in the first line should read $\frac{0}{100}$, thus recording 100 "*lesses*" and no "*mores*"; and the second line should read all $\frac{100}{0}$, or 100 "*mores*" and no "*lesses*"; and the sum of the two lines should average (in the right-hand or average columns) $\frac{100}{100}$. Just in so far as the fractions depart from the above formula, they indicate errors of judgment, and the direction in which they depart indicates the peculiarity of these errors; this is the peculiarity which we are in search of. For example, the fraction in the upper left-hand corner shows, that of 100 comparisons of a distance 1 centimeter, pressed on the forehead vertically, with a distance .9 centimeters, pressed horizontally immediately after, the latter was judged correctly to be shorter than the former 80.75 times, and incorrectly to be longer 19.25 times. The parallel fraction, next below, shows that of 100 applications of .9 centimeters horizontally, followed

by 1 centimeter, vertically, the latter was judged correctly to be longer than the former 32.5 times, and wrongly judged to be shorter 67.5 times. If we now add these two fractions we have $\frac{51.75}{148.25}$, and this indicates, in so far as it goes, a tendency on the forehead for distances to seem longer when pressed vertically than when pressed horizontally. Of course the sum totals, which foot up the average columns, express this tendency more generally; for instance, the final fraction, for the forehead, shows for all the categories and degrees of difference worked with, that, on the average, the vertical distances seem longer than the horizontal ones by a ratio of $\frac{1022}{1978}$, or about twice out of three times.

Of course the above ratios do not express the *amount* of error by which the distance is over judged, or foreshortened. Some idea of this amount may, however, be gained by casting the eye along the line of fractions from left to right, while observing the amount of difference between the compared distances as indicated in the headings of the several columns. Thus it is easily seen that this difference for the first column is 1 millimeter, for the second column 2 millimeters, the next 3, the next 4, and the next 5, thus increasing from left to right through those columns wherein the standard distance is always 1 centimeter. The ratios of the first line of fractions should increase, and those of the second line

decrease from left to right through each set of 5 columns. What this expresses, in a general way, under the Psychophysic Law, as to the amount of error by which vertical distances seem longer than horizontal ones, is obvious, though in itself the amount cannot, from these figures, be exactly determined.

Table 28. Experiment F.—Comparing

Vertical Distances.		Order of Comparison.	1	1	1	1	1	2	2
Horizontal Distances.			.9	.8	.7	.6	.5	1.9	1.8
FOREHEAD.	Averages.	$\frac{V}{H}$	19.25 80.75	15.75 84.25	13.25 86.75	14.0 86.0	12.75 87.25	22.75 87.25	26.25 73.75
		$\frac{H}{V}$	32.5 67.5	31.75 68.25	30.5 69.5	39.5 61.5	46.75 53.25	35.25 64.75	44.5 55.5
FOREARM.	Averages.	$\frac{V}{H}$	46.5 43.5	41.5 58.5	30.25 69.75	25.75 74.25	20.75 79.25	66.75 33.25	64.50 35.5
		$\frac{H}{V}$	49.25 50.75	47.0 53.0	56.0 44.0	53.5 46.5	55.25 44.75	62.0 38.0	59.5 40.5
ABDOMEN.	Averages.	$\frac{V}{H}$	33.25 66.75	33.75 66.25	31.5 68.5	38.0 72.0	31.5 68.5	45.75 54.25	45.5 54.5
		$\frac{H}{V}$	27.75 72.25	37.75 62.25	45.75 54.25	33.0 67.0	37.0 63.0	48.25 51.75	43.75 56.25

horizontal and vertical distances.

2	2	2	3	3	3	3	3	Totals.	Sum Totals.
1.7	1.6	1.5	2.9	2.8	2.7	2.6	2.5		
20.75 79.25 45.25 54.75	14.25 85.75 64.0 36.0	12.25 87.75 61.75 38.25	32.0 68.0 49.75 50.25	38.5 61.5 43.0 57.0	32.75 67.25 56.5 43.5	27.5 72.5 60.0 40.0	21.0 79.0 58.25 41.75	323 1177 699 801	1022 1978
62.5 37.5 66.5 33.5	60.5 39.5 65.0 35.0	48.5 51.5 68.0 32.0	75.25 24.75 77.0 23.0	72.25 27.75 79.25 20.75	82.0 18.0 77.25 22.75	71.0 29.0 81.25 18.75	68.0 32.0 79.25 20.75	836 664 976 524	1812 1188
37.5 62.5 39.5 60.5	36.25 63.75 49.75 50.25	33.75 66.25 43.0 57.0	47.75 52.25 55.75 44.25	47.25 52.75 62.75 37.25	48.0 52.0 62.5 37.5	52.25 47.75 64.5 35.5	57.5 42.5 66.75 33.25	614 886 718 782	1332 1668

A STUDY OF THE RESULTS.

NUMBER.

§ 1. The two upper blocks of figures, of Tables 1 to 7, relate to the "number judgments" obtained in Experiment A. Examination of these figures discloses that their distribution in every block is governed by certain main laws, all holding good throughout, though with variable force in their relative manifestations under the different conditions of the experiments. We have to note these laws and to inquire their meaning.

The first is a law of chance, imposed by the methods of the experiment. We note that in the second block of figures, through all the tables, the values in the "II pin" or left-hand columns are invariably plus, and those in the right-hand or "V pin" column are always minus. The reason for this is obvious when I explain, that the subject always knew what categories were being used in the experiments.[1] In the present "number judgments" he knew there could never be less than two pins, nor more than five. Consequently there could

[1] It is best to explain these to the subject from the first, as it is impossible to keep him from forming notions about them during the course of the experiments.

never be any minus errors in the number judgments of "II pins," and never any plus errors in those of "V pins." By mathematical calculation we could theoretically deduct the effects of chance from our work; but as we should then have a set of figures but little if any more significant for our purpose than those already given, I have neglected such theoretical calculations.[1]

Other things being equal, the less the average amount of error made in judging the pins, the greater number of times per hundred should the pins be judged correctly. Consequently the distribution of the figures in the upper row of blocks ought always to stand in inverse ratio to that of the corresponding figures of the blocks below. Examination will show this to be the case.

§ 2. If we again study the two upper blocks in our first seven tables, we shall note the second of our three laws of distribution. It may be stated as follows: The longer the distance, the more accurate the judgments.

[1] A few points, however, may well be borne in mind. Namely: that in the second row of blocks, the effect of chance is to make the values in the "II pin" and in the "III pin" columns, respectively, as much too great (+) as in the "IV pin" and "V pin" columns they are too small (—); and also that, respectively, the + and — errors of III and IV ought to be less than the like errors of II and V. If all the errors were solely due to chance, then all the values of II and III should be +, and all of IV and V should be —, and II and III should, respectively, balance IV and V. All deviations from such distribution must indicate the influence of other laws yet to be determined.

This law appears simple enough so long as we study alone the judgments made of II pins, and without asking *why* any such law ought to hold good. Practically, throughout all the blocks and tables, the "II pin" columns show regularly increasing accuracy as the distance increases from 1 to 3 cm.; that is, downward in these columns. Moreover, the "III pin" columns show openly, in general, a tendency to follow this same law. We may note, however, that usually, *throughout the shorter distances*, these "III pin" judgments incline to follow an opposite course; beginning at the top of these columns the accuracy of judgment appears to decrease, till a certain length of distance is reached (differing according to the region of body studied), whence onward, with increasing distance, the figures follow the law at present in hand as regularly as do the judgments of II pins throughout. Already this regularity of exception to our present law foreshadows the coöperation of a third law. But we feel much more the need of some such further principle to account for our results, as we come to examine columns IV and V. Here we appear to have, completely reversed, the very common law of experience, which certainly holds good for two pins, and if we accepted the mere empirical evidence of these figures, we should conclude that, in judging four and five pins, the effect of spreading them further apart decreased

the accuracy of our judgments. This brings us to our Third Law.

§ 3. If we say that, with decreasing distance, uncertainty of judgment increases, we shall merely be stating our old Law Two in a new form. But "increasing uncertainty" means increasing liability to spread our judgments over categories other than the correct one; it means increasing tendency for the mind to pitch its judgments upon its possible categories, rather than upon the right particular one; to depart from highly specialized, accurate, and fixed habit, to less definite, and less rigidly developed habit.

If now, according to Law Two, with decreasing distance we have increasing uncertainty, then, by Law Three, with decreasing distance we have a tendency to spread the increasing uncertainty more and more over the wider field of possible judgments.

By "possible" we must not mean, as limited to the four number categories of pins in our experiment, but possible by our whole nature. The whole range of numerical categories, which is possible in this larger sense, is very great, and the relaxation of accuracy or of particular habits does not take place in equal proportion toward all. For the four number categories used in our experiments, the uncertain judgments continually drift toward higher numerical categories. The lower the numerical category the stronger is this tend-

ency. Or at least, the drift of errors being in a general direction, we find as the outer impression is moved in that direction, the less becomes the error which is due to the drift of uncertainty. It is probable that if the numerical categories of our experiment extended high enough, we should find a place where there would be no tendency for uncertain judgments to drift in any single direction. The reason for all this we shall come to presently, but we may now throw our Third Law into its empirical form as follows: The lower the numerical category, the stronger is the tendency of the uncertain judgments to drift toward overestimation.

§ 4. Having found our three laws we will now examine our tables. We shall need, here, to follow in detail but a single example, and simply because they are longer than some of the others, we will take Blocks 65 and 70 in Table 4. Beginning in the upper left-hand corner of 65 we find that, for the region here studied, two pins, when separated by a distance of 1 cm., are judged correctly only seven times out of a hundred. From the figure 7 vertically downward, the numbers increase pretty regularly till at 5 cm. two pins are judged correctly seventy-nine times. All this plainly, by reason of Law Two. Going back to the same figure 7, we see the numbers increasing horizontally to the right, till, with the shortest distance

category of 1 cm. remaining constant, we find V pins judged correctly fifty-five times. All this, however, as I believe, wholly from the "drift of error," and not as in column II through increased functional accuracy of judgment. Merely, here, Law Three overbalances Law Two. As we run down the left-hand column V, we see the number decrease pretty regularly from 55 to 28. This indicates no contradiction or suspension of Law Two, but by Law Three, with increasing distance should go decreasing drift of error; that is, less tendency of the uncertain judgments toward overestimation. If we look below, in Block 70, at the corresponding figures to the last above-mentioned, it would seem at first sight absurdly incorrect to assert overestimation for these judgments at all, for all the amounts of error are given as minus. But plainly this is explained by Law One. By the terms of the experiment there could be no judgment greater than five pins. But the "drift of error," by Law Three, was active all the time, even as against these terms, and made the values appearing here as relatively minus, really greater; that is, less minus than they otherwise would have been. As evidence that the natural drift of error for the whole range of numerical judgments from II to V inclusive is positively upward, it is to be noted that the average error for all the distances, if calculated, would be plus; that the average for all distances is + 11.44; and that

a roughly estimated deduction of the "chance" values from the various columns, as suggested in our discussion of Law One, would leave an unmistakable indication that the real tendency in this range is throughout toward overestimation.

The blocks above examined are typical, and we may observe of the number judgments throughout our tables, that in column II the obviously controlling influence is Law Two: with increasing distance goes increasing accuracy. In column V the obvious influence is Law Three: with increasing distance goes decreasing uncertainty; therefore decreasing drift toward overestimation; therefore decreasing correction of functional inaccuracy by "local drift"; therefore fewer correct judgments—the influence of Law Two outweighing that of Law Three, the latter remaining active all the while. In column IV, as it should be, Law Three is less powerful than in V, but remains the obvious influence. In column III, as it should be, Law Three is strongest in the shorter distances, and with decreasing force is dominant up to a distance (about 2.5 cm. in the figures for the abdomen) when its influence gives way to that of Law Two, which holds sway thence upward. The influence of Law One we have already made sufficiently obvious.

§ 5. Having enabled the reader to study our tables for himself, I shall now dare to ask him to consider

their content and their laws in the light of a fundamental hypothesis as to the formation of numerical judgments in general.[1]

Attacking our Law Two, to discover *why* with increasing distance there should go increasing accuracy of judgment, we must first ask why the simultaneous stimulation of two points of skin lying under two pin-points, situate a proper distance apart, should ever give rise to the conception of duality at all.

The reason for this is somewhat as follows. In the first place these two areas must previously, sometime in life, have been stimulated the one after the other in immediate succession. The conception of duality in general is a particular mental state or process which is the result of such a shock; it is the feeling of such a shock. As such it falls under all the laws of Association and of Memory which govern other conceptions, notions, mental states, and processes. As such, again, it is subject to all the effects of habit which, going back a step further, govern these above laws; and in turn the particular habits, which control the associative and perceptive activities of any conception of duality for any particular pair of points or areas in any given region, depend still more fundamentally upon

[1] I find the main thoughts of this hypothesis best stated in Professor James' *Principles of Psychology* (vol. I, chap. xi, in particular, pages 487, 488, 498).

the average run of experience common between these two points or areas. The final habit is the resultant average of all the past habits. On the whole, during life these particular areas have not been as frequently stimulated simultaneously as successively; consequently the successive "number mode" which is the conception of duality, has become more strongly established as between these two points, than has the mode native to simultaneous stimulations, namely, the conception of unity. We could, perhaps, by refined means, stimulate the total nerve ends of even these two pin-point areas in some plural form of succession. That is, we could divide them into three, four, or any number of separate groups and then stimulate these groups in succession. And if such a practice prevailed through life above all other modes of stimulating these areas collectively, then by our hypothesis, the simultaneous pressure of two pins upon these separate areas would give us the numerical conception corresponding to that mode of succession, rather than as now to the dual mode. Why, therefore, the pressure of two pins on separate areas commonly give us a conception of their duality, is not so much that the particular tools of stimulation then and there used are two pins, as that between those precise areas taken collectively the mode of stimulation, which, on the whole, through life has prevailed and set up its particular habit of mental reaction, has been the mode of dual succession above all others.

This being so, the explanation of our Law Two (that with increasing distance the habits of plural numerical judgments become more accurate) is easily reached. In very small areas the nerve ends collectively are more frequently stimulated together than separately. The habit of unity prevails over all other numerical habits. Hence stimulation of such areas is most likely to give rise to the numerical conception of "one thing." This will hold good even though the tools of stimulation be the same two pins which, when set further apart, will invariably give rise to the notion of their being two; if the two pins be set too near together they will ordinarily be judged as "one." When, now, we come to spread this spacing toward wider distances, we depart from conditions where the unitary habit is strong toward those where this habit is less strong, and where the habit of duality begins to be its rival. As we go on widening we find the former continually weakening till it fails entirely, and the latter growing more and more strong till its judgments approximate absolute certainty and accuracy.

Our Law Two, therefore, in so far as it relates to our judgments of two pins, is but an expression of the fundamental fact that the further two points are separated on the skin, the more confirmed become the resultant habits of experience, relative to those points, in favor of the dual mode of reaction over and above all other modes of numerical reaction.

The law holds equally good and expresses precisely similar facts in higher numerical judgments. Judgments of "three," and of "four," are particular mental states or conceptions, based upon series of three, and of four successive sense impressions, in a manner strictly analogous to that in which judgments of "two" are formed. Other things being equal, the further apart any x number of separate points or areas of skin shall be, the more through life are all those stimulations or impressions, which bring all those points into collective relationship with one another, likely to fall into a series of x successive impressions rather than into any other particular combination collectively of those several points. The influence of this truth upon our mental habits works as a factor in the formation of the judgments of one numerical category as certainly as in the formation of those of another. But while it may be an unmixed influence in one category—such as we discovered it to be in the "II Pin" columns of our tables—it may be mixed with other influences in judgments of other categories, such as, for instance, those of columns IV and V. It remains to glance at some of these other influences, in the light of our general hypothesis.

§ 6. One of these is the influence, under Law Three, by reason of which the drift of errors in uncertain judgments (at least in those for the categories used in our experiments) is constantly toward overestimation.

We have pointed out that in departing from habitual reaction in a single way, we sink to a looser and wider range of reactions. If the bond of connection or of associative habit for the uncertain sense impressions were equally strong toward all the possible numerical reactions, then it would be easy to see why, for judgments of the lower numbers, the average drift of error would constantly be toward over-estimation. It would be a mere matter of mathematical chance, or, as we might say here, of psychological chance. The drift being equal, the average drift for the lower numbers would necessarily be upward. How far this serves to explain the actual drift of the errors of uncertainty in our experiments, or whether it is a correct explanation at all, I cannot at present with any certainty decide. Personally, however, I incline to look upon this drift as the expression of a loose and inexact general habit of the whole brain, or of a large sphere of it, to act most strongly in the direction of the general average of the entire range of experiences embraced in that general habit; this rather than to conceive of the many errors, actually committed in uncertainty, as so many accidental reactions in several loosely incitable but definitely directed habits.

§ 7. Further evidence for our hypothesis appears when we compare the results obtained upon one region of the body with those of another, but for reasons that

will become evident I will postpone considering this matter.

§ 8. With reference to Tables 5 and 6, both refer to the same region of skin — the forearm. The difference between the method pursued in the regular experiments upon the forearm (Table 3) and that which gave us Table 5 is, that in the former the row of pins was rocked lengthwise upon the skin, and in the latter all the pins were pressed on evenly and at once. The former gave series of impressions precisely like those which originally gave rise to our numerical judgments. They *are* such original impressions as our habits of numerical judgment at first hand are founded upon. The latter method gave us only simultaneous impressions; these were no longer impressions like the original impressions; were not the old successions happening over again; and the judgments which followed were only weakened imitations of former judgments awakened at second-hand through memory and association. Now, since these latter judgments depend more upon memory than do the former, they must be more uncertain than judgments based upon successive impressions. But with increased uncertainty should go more marked exhibition of Law Three; and if in our tables we discover this, we should count it as confirmatory of our general hypothesis as outlined from the beginning of our paper.

Turning to Block 83 of Table 5 we do observe just such an increased influence of Law Three in proportion to the relative influence of Law Two, as we have spoken of. Comparing these figures with the corresponding ones for our "regular" experiment on the forearm (Block 45, Table 3), we find unmistakable evidence of greater uncertainty and of the distribution of the consequent errors according to the law which we have laid down and given the reasons for. In column II, where the effect of Law Two always is most obvious, we see the accuracy of judgment reduced by some 30 to 50 per cent., while in column V, where Law Three is most evident, we find an increase of its influence about the same in amount. Moreover, as the method of applying the pins was changed alike for the whole scale of distances, so the results show a tolerably equal amount of change throughout the block; that is, the increased drift toward over-estimation is a pretty equal one throughout. All the numbers in column V are approximately as much increased as those in column II are decreased. In short, throughout, with a like change of method we see a like change due to Law Three.

§ 9. For the results shown in Table 6, not only were the pins applied evenly without rocking but, by the effort of will, the attention was confined strictly to the number of pin-points clearly felt. Practically, this means that the mind was not permitted to range up and

down the line of pins, as it sat on the skin, "listening" here and there as to whether a pin really was felt then or not, and, by a continuation of this process, coupled with a knowledge which the subject always had of all the categories which were being used, to reckon out just what combination of pins and distance was at the moment being applied. I say the mind was not permitted to range up and down. This is but saying that the fundamental process which is the basis of Law Three was here not permitted to play. The memory process was shut out, and consequently the drift toward over-estimation was shut out.

Not only this, but I am inclined to think that the actual results reveal to us, in a strikingly significant manner, the working of memory in reviving through *simultaneous* impressions the numerical judgments which originally, at least, were the effects of successive impressions. The "ranging up and down of the mind" in search of the proper category is much like actually playing over again in imaginary processes, the actual successions of the original events; and the effects of inhibiting this "ranging," as shown in Table 6, are so surprising as to emphasize the question as to whether or not such imaginary successions, in some form, perhaps almost infinitely compressed, are not what really happen, in the formation of all numerical judgment from a *simultaneous* impression, and whether, therefore,

they are not absolutely necessary to the formation of such judgments? Under the influence of the inhibition of such processes, column V of Table 6 shows, in some places, an absolute lack of correct judgments of the higher categories — those which would require the greatest amount of this imaginary play — and shows but a very small number of such judgments anywhere throughout the column. Nothing I have said, however, must be mistaken for a premature inclination to decide this matter.[1]

DISTANCE.

§ 10. When we draw a pencil-point along the skin we stimulate successively the nerve ends lying in the line drawn. In essential nature such an event is no different from those primitive occurrences which, according to our hypothesis, give origin to our number judgments. They are both based on serial impressions. The difference between the number judgments and the distance judgments lies chiefly in the nature of the successions which characterize each. In the former the terms of the series are comparatively few, the successions are sharply marked-off, and slow; so slow and marked that we note and *count* them — successively give

[1] I am sure that our experiments here are capable of teaching us an important lesson as to the intimate nature of the processes of attention in the formation of judgments in general.

names to them. Thus: one — two; or one — two — three. When a line is drawn along the skin a myriad of nerve ends are stimulated in relatively rapid and unmarked successions; so rapid and unbroken that we do not note the separate terms, nor individually count them. The difference is that in one case we say, "one — two — three," and in the other, "so ma-a-a-a-any" or "so fa-a-a-a-a-a-ar."

Each specific length, however, of the distance series has a nature of its own which is the basis of each one of our specific categories of distance judgment (our ideas of particular distance), in the same way that the number series each have a particular length or number of terms which is the basis of each category of number judgments.

§ 11. We have said that, given a definite set or arrangement of nerve ends, until this set, some time in life, be broken up and its parts be first stimulated in some sort of succession, any sort of simultaneous stimulation of this particular set will not arouse any sort of plural category whatever. We now say that it will arouse no conception of distance whatever. Any set of nerve ends which has never been stimulated except in complete simultaneity will give us the same sort of mental response or experience when distributed over the surface of the skin in a compact bunch, as when distributed in any sort of lineal arrangement.

§ 12. Yet it is the fixed lineal or spatial arrangement of our dermal nerve ends that determines our particular distance and space conceptions regarding them. This happens because it is the fixedness of each particular arrangement that determines what manner of serial stimulation through life shall most frequently fall to the lot of that collective group of nerve ends. If two nerve ends are permanently located immediately beside each other they are likely to be stimulated more times during life simultaneously than successively; and consequently, upon simultaneous stimulation, are more likely not to arouse any notion of distance between these two points than to arouse such. If two nerves are fixed widely apart they are more likely than otherwise, in the whole of life, to be affected by various moving impressions which shall first stimulate one nerve at a given point in the series of impressions and the other nerve at another point in the series. Consequently it is more likely than not that some sort of distance category will be developed and attached to the collective stimulation of these two separate points.

§ 13. Every pair of separate points is likely to be stimulated by all sorts of lineal impressions moving first through one point and then through the other. This, in the same way that it is possible to draw all sorts of lines (straight, broken, and curved) through any two points of our skin. The question then arises how

any particular distance category, corresponding to some particular length of moving series, comes to be so joined to any two particular points that we commonly judge them to be a definite, actual distance apart?

This is not difficult to answer when we recall that the kind of memory category that is awakened by the simultaneous stimulation of any definite combination of nerve ends, is based on the dominant and average habit which is the resultant of the experiences which have most frequently combined that particular collection of nerve ends. With reference to the skin, it is evident that right-line movements are likely to prevail between points separated by a few centimeters (as in our experiments) far and away above any other particular form of lineal movement.

§ 14. But the rate of drawing a line may be infinitely variable, and the time element of the series is the most important thing of all, according to our hypothesis, as the primitive basis of distance measurement. Here, again, we see that the single definite habit, which finally results from the particular modifications of each one of the infinite number of infinitely varied time series experienced through life between every pair of dermal points, solves the difficulty. The average of such an infinity of time experiences for any pair of points would, other things being equal, be proportional to the actual, fixed right-line distance between the points. Consequently,

in proportion as the resultant or prevailing habit is a fixed and accurate one does its mental judgment or perception accurately represent or correspond to the actual right-line distance.

§ 15. At this point many difficulties arise if we inquire as to the intimate and specific nature of our different distance perceptions. All that we have said supposes them to be based upon reawakened time series of correspondingly specific length or nature. Yet we surely must reject the notion that our hasty judgments of different distances are always of the same absolute and specific time lengths, all in due proportions. But if not, how do they preserve any sort of proportions between themselves, duly representative of actual outer differences? This leads us to one of the most obscure regions of psychology.

To me the following hypothesis seems both more clear and more justifiable than the average psychological hypothesis of modern text books relative to the intrinsic nature of a judgment. It is the connective or associative function of any mental processes or habit that is of importance in the formation of accurate thoughts and judgments, rather than the nature of its content. If the function is accurate and specific, the judgment will be accurate and specific. The function of the specific perception or judgment is, to make the proper connection between the outer event or impres-

sion and certain following thoughts, perceptions, or associations *about* that event or impression. If the proper specific connection is made, it makes no, or little, difference to the accuracy of the thinking, what the specific nature of the mental content of the judgmental in itself may be, either qualitatively or in absolute time-duration. Suppose the perceptive connection is to be with the motor idea which incites us to say, "one centimeter." The connective activity may occupy an absolutely longer or shorter time, and this make no difference provided the nature of the process is such that the proper motor-idea is eventually incited. This being so, I think we may easily conceive how that, though our various specific judgments are all *based* upon habits which are the resultants and the correspondents of time series of relatively different absolute lengths, they yet may not themselves occupy absolute intervals all proportionally different, nor their conscious content be of any given specific phenomenal nature. Indeed we may easily conceive how the really decisive and distinctive link should be wholly an unconscious mental process. If we take our stand on the Summation Theory, we can conceive how the specific *intensive* force or constituency of the differently summated series might determine the proper connection independently of specific time durations. Or we may with plausibility conceive of some specific anatomical arrangement of

brain parts, correspondent to each specific judgment, whose liability to be affected as a whole, depended upon specific series of stimulations of absolute length, and yet whose subsequent activity as specific wholes in memory should not necessarily occupy always the same absolute time-rhythms. Or, better still, perhaps we may conceive of a combination of intensive, anatomical, and spatial distributions which shall mediate the activities correspondent to the correct specific judgments and make accurate connection thereby with the proper associative thoughts, yet do this quite independently of an absolute time performance which should be preserved constant for every repetition of the judgment.

§ 16. With so much of our hypothesis before us we may now come nearer the laws which specially govern our tables. To begin with, we may note that in the main all we have said about "chance" must hold good as much for the two lower rows of blocks in our tables —those which relate to the distance judgments—as it does for the two upper rows, which relate to the number judgments.

§ 17. According to Law Two the numerical judgments were the more accurate, the greater was the distance. Under what we may call the same law, we shall now find the accuracy of the distance judgments also, as a rule, increasing with the actual distance. But the reasons for this law are neither

so plain nor so simple as previously. Since our perception of the distance between two points of skin is based, primitively, upon the predominance of movements in the right-line joining those points over movements in any other particular line or combination of lines; and since the greater the distance the less should be this predominance, therefore, were these the only principles determining Law Two for distance, we ought to find increasing inaccuracy with increasing distance. It is probable that this principle has its proper influence, primitively, in developing our notions of particular distances everywhere, and very likely there are lengths of distance for certain stretches or regions of skin where the principle is a predominating one; but within the short distances and regions investigated by us, other principles come in which counteract and far outweigh it.

§ 18. One of these other principles is similar to the one which explained Law Two under "Number." We may describe it as follows: Our memories concerning the distance between two points are based on movements between these points; the longer the distance, the more are the two points disassociated by these movements; that is, the more do these movement-habits weigh against the tendency for the two points to fuse spatially in memory; they tend so to fuse in proportion as the points are habitually combined simultaneously. There-

fore, in forming the prevailing memory-habit, the disassociation of movement is always opposed to the influence of simultaneous combination. Thus Law Two in distance, or at least within short distances, formulates, as in number, the increasing tendency of the influences of the successive experiences of life to prevail with increasing actual distance over the influence of the simultaneous experiences of life. Put into every-day language, this is but saying that we are more likely to judge the distance between two points accurately, the more capable we are of forming a distinct conception of their separateness.

Put more technically: the further apart two points of skin are, not only is our conception of their separateness heightened through disassociation due to the movements between those single points, but also it is heightened through the growing differentiation in the grouping of the local associations *around* each point. If the distance increase enough, the two points will habitually fall into strikingly incongruous groups. For instance, the point of the toe and the back of the head. Now, it is plain to any one having much experience as a subject in the experiment we are discussing, that our judgments are not confined to data based on the precise line joining the terminal pins. Rather, we form a notion of the separate and particular *region* where we find one pin to be, then another notion of the region of the other pin,

and we then say that one region is "so far" from the other. This being so, we see at once that the particular judgment of distance which we finally render, is not based on the simple disassociation-force of the single line between the two pins, but by the whole disassociation-force of the two "region groupings"—the whole force of the region's associations to fall into distinct and separate groupings rather than to fuse into an unseparated single spot.

§ 19. Still another principle works in favor of Law Two. Our voluntary measurements of shortest-distance are always in straight lines. When we *train* our judgments of distances, we are always training our right-line memories. We know how great are the results of training, and how great must be the influence through life of joining the right-line memory to, and weaving it into, the concept of measuring.

§ 20. I think it should now be clear how increasing distance works to increase the accuracy of estimating the distance between points. The greater the distance, the sharper and stronger will be our conception of the separateness of the two regions. The sharper and stronger this conception of the regions, the more highly developed in connection therewith will be the memory effects of the voluntary measuring of the points. The voluntary measurements will all be based on right-line experiences. The three concepts — of the regions, of

the measuring, and of the right-line — will fuse, and act as a whole; the resulting judgment will be strong, clear, and accurate, in proportion to the strength, clearness, and accuracy of the united conceptions. As two of the conceptions are increasingly sharp and clear with increasing distance, so does the total resultant judgment increase in accuracy with increasing distance.

It remains to be said of Law Two, that for very short, or sub-threshold distances, we should expect to find its effects very weak; for them all the elements of the law would be very poorly developed. The conception of the separateness of the points would be faint; the skill of measuring such unusual distances would be scanty; and the immediate impressions would tend to group into "spots" rather than to revive the serial memories of right-lines. The judgments would be uncertain, and would, consequently, fall under the influence of the laws of uncertainty. This brings us to the law of uncertainty, which we called Law Three.

§ 21. Law Three, in the number-judgments, formulated the drift of the errors of uncertainty. It will do the same here. As before, the drift will be toward the average of the possible categories. But the spatial categories are more variously conditioned for different regions of the skin, than are the distance-categories for the same regions. That is, the character of the spatial experiences depend far more on the shape, area, and

contour of a particular area of skin, or member of the body, than do the numerical experiences. For instance, the distance series native to the tip of the tongue would never be long, while the number series might run as high there as anywhere. We shall then expect to find over-estimates and under-estimates in any fixed scale of categories of distance, like those of our tables, to be greatly variable between different regions of skin. While we shall find ourselves obliged to examine each region more carefully by itself, to determine the precise influence of Law Two in each case, yet it will be just this lawful variableness which will be of significance to us when, as we propose, we come to test the truth of the Genetic Hypothesis, as a whole, by comparative studies.

§ 22. Beside the above laws, we shall discover for distance-judgments still another law, which played no part in number-judgments. We will call it Law Four, and it may be stated as follows, *i.e.*, The greater the number of pins, the shorter will be the estimated distance. And since the right-line distance will be both the shortest distance and the correct distance, we may say that, other things being equal, by Law Four: The greater the number of pins, the more accurate should be the distance-judgment.

This is to be accounted for as follows: Our judgments are based upon the memory habits joined to particular

nerve-ends. Also, our distance-judgments are based upon right-line movements between points. Now it is plain that, in these original movements, every nerve lying in the line of any movement would be as much joined to the resultant memory effects of that movement, other things being equal, as would any other nerve lying in that line. Consequently, although we shall discover other reasons than the above why that particular distance-memory, as a whole, becomes more joined to the end-points of the line than to intermediate points, we still may see from the above reasons why each intermediate point is very intimately joined with that particular memory. This being so, it is easy to see why every additional pin introduced between the two end pins in our line of pins should be an additional stimulant to the revival of the proper perception and judgment. Each pin in the right line is a guide toward the distance perception, being based on the right-line memory, rather than on the possible memory of innumerable other lines. Consequently, the greater the number of pins in our experiment, the more accurate should the judgments of distance be.[1]

[1] The reason why some other category of distance rises upon simultaneous stimulation of two intermediate points of an original right-line movement (namely, that of the shorter distance between the intermediate points, rather than the original category) is plainly not because the longer category has *no* strength, but because the shorter category has the stronger and prevailing

§ 23. Having explained our laws we will now examine our tables. Turning for illustration to Blocks 75 and 80 of Table 4, which contain the figures corresponding to those already used in illustrating the laws of numerical judgments, we first observe in Block 80 that all the values in the upper horizontal line are plus, and all of those in the lowest horizontal line (above the averages) are minus. This is the effect of "chance," and Law One, which, it will be observed, now works in even horizontal lines, up and down, from top to bottom of the block, instead of right and left, as before, in the number-judgments. It would be corrected by subtracting a maximum amount from the (+) values in the top line, and adding a like amount to the (—) values in the bottom line, and grading proportional corrections

strength, as between these two points. And the reason why the several categories, corresponding to each distance between each intermediate point of pins, does not rise to perception, under simultaneous impression of the whole line of pins, is not because there is no tendency for the several shorter categories to rise, but because, again, the tendency for the single longer category is the prevailing category. Why the longer one should be stronger than the shorter ones is plain, if we remember that each intermediate pin would have some tendency to call up the outside category, while the pins outside of each intermediate pair would not have equal tendency to call up the intermediate category. Why we can think alone in the one strongest category, and cannot think in all the categories at the same time, lies, very probably, somewhat in the fact that the same brain parts are likely to be demanded simultaneously in the several categories, and can act only in the one line of strongest tendency.

from these extreme categories toward the middle category of 3 cm., where the influence of the law is negative and zero.

§ 24. We next observe the effects of Law Two. By this, all the judgments of distances above the threshold distances should increase in accuracy with increase of distance. Assuming the threshold for this region to be about 3.5 cm., we see the number of correct judgments increasing regularly with increase of distance throughout the remainder of the blocks. The average in Block 75, for 3.5 cm. is 20.5; for 5 cm. is 40.2. That the law begins to have effect even in the short category of 1.5 cm. is obvious from the figures.

§ 25. Law Three shows a slight tendency toward over-estimation in the region of the abdomen throughout. The total average in Block 80 is +3.46, and the average for the middle category of 3 cm. is +5.6. The tendency is, however, so light, that under the shortening influence of Law Four, the actual tendency is toward under-estimation for V pins, even in the short distances, where by effect of Law One, the actual judgments should show plus values.

§ 26. The effects of Law Four are most obvious in the 1-cm. judgments, and in the horizontal averages of Block 75, and they show markedly in Block 80 throughout. The effects are exhibited as increasing accuracy from left to right, horizontally, across the four columns

of pins. Thus in Block 75, we read for the 1-cm. judgments, 42, 53, 66, 78; and for the averages, 30.6, 30.8, 31.3, 31.6; and the decrease of the amounts of error in Block 80 may be illustrated by the averages, which read + 10.3, + 6.0, + .4, — 2.6.

§ 27. To test the united influence of our various laws we will now examine Blocks 75 and 80 more intimately. Studying the top line of Block 80, we see as follows: Law One makes all the values more plus than actually they should be. Law Two shows a maximum of uncertainty for this, the shortest category of distance; the average error, + 26.0, would be the greatest for any of the distances, even after making corrections for Law One. Law Three shows a drift of error proportionate both to the uncertainties due to Law Two, and to the uncertainties due to Law Four. Proportionate to Law Two, there is over-estimation holding good for the average of all the minimal distance-judgments, the average error being + 26.0, — a sum that indicates over-estimation after correction for Law One. Proportionate to Law Four the drift of error is greatest where the uncertainty by Law Four is greatest, *i.e.*, where the pins are fewest, and decreasing as the pins increase from left to right; thus + 37.0, + 29.8, + 21.0, + 16.1. Law Four shows a shortening of the judgments with increase of pins; this is shown in the four numbers last quoted. Of course the top line of Block 75 would

show effects corresponding to the above, if correspondingly analyzed.

Next, examining the figures of column V in both blocks, we find: Law One making the judgments too long in the short distances and too short in the long distances. Law Two makes the judgments more accurate with increase of distance. This is obvious in the lower half of Block 75, where the number of correct judgments increases with the tolerable regularity of 20, 23, 20, 39 for the distances 3.5–5 cm. The real effects corresponding to these, in Block 80, are obscured by reason of the fact that the minus quantities due to Law One increase in this column from the middle downward, at a ratio greater than that by which the minus values, resulting from drift of error (under-estimation in this V-pin column) decrease through increase of accuracy due to Law Two. Make correction of Law One, and Law Two is very evident. The real effects of Law Two in the upper halves of the two blocks is obscured in a like manner. That is, in the upper half of Block 80, we have minus values even against the influence of Law One, which theoretically should give stronger plus values; consequently, here there must be under-estimation by Law Three, heightened through the influence of the high number of pins, *i.e.*, by Law Four. Now, since the plus influence of Law One falls off, with increasing distance, faster than do the united minus

values of Laws Three and Four, we therefore have the apparent contradiction of Law Two; the contradiction, however, being only apparent, and due, as we have seen, to the compensating influences of the other laws. The influences of Laws Three and Four are, from the foregoing, sufficiently plain, and the entire distribution of the figures in column V should now also be clear, as displaying and agreeing with the united influences of our several laws.

Examining the lower horizontal line (above the averages) to test our laws in the higher distances, we find as follows: Law One gives minus errors throughout. Law Two gives maximum accuracy for the longest distance; as is evident from the large number of correct judgments shown in Block 75, and the small average error that would remain after correcting the minus-constant of Law One. Law Three averaged for the four columns of pins, shows a slight tendency to over-estimation at the distance of 5 cm., Block 80. Law Four makes this over-estimation more pronounced in the low-number columns, and shortens it perhaps to actual under-estimation in the right-hand or "V-pin" column. As the result of the combined influence of the four laws, the effects of Law Four are, in Block 75, apparently contradicted, that is, the actual number of correct judgments decrease from left to right (48, 44, 40, 39); but it will be easily understood from the foregoing that this

appearance is but the effects of the various compensations which we have last above described and illustrated from the last line of Block 75.

The distribution in the three remaining vertical columns are so similar to that of column V that now, having both followed in detail the three sides of our blocks, and also examined the general influence of each law upon each block as a whole, I think the detail of the distribution in all the columns throughout should be perfectly plain to any one upon due examination.

Our sample blocks, therefore, we find conform to our laws. This conformation will have weight in support of our main hypothesis, just in proportion as, with integrity, it may be shown to be representative of a wider conformation extending throughout the large body of our experiments. Manifestly it would be impossible within any reasonable limits of publication, to go through a detailed examination similar to the above, explicitly demonstrating the course of our laws and of our main hypothesis throughout each of the 123 blocks of figures presenting the extensive and arduous series of investigations classed together in this paper as Experiment A. Still less would it be possible to extend such a demonstration throughout the 365 blocks of Experiments A to F. This each student, according to his interest in the matter, must do for himself. But I assert with the confidence of long and careful study

that such an examination results in undeviating and constant accumulation of evidence of the integrity of our laws wherever they are implicated, and of the soundness everywhere of the main lines of reasoning upon which they have been founded. Where at first there may appear to be contradictions, we shall discover by closer study, and especially by the comparative studies already foreshadowed, that these are but the apparent exceptions which, when understood, all the more abundantly substantiate the general truths.

§ 28. Before leaving the present discussion of distance a few things remain to be said in connection with the special tables of Experiment A.

Table 5.—Here the pins were pressed evenly without rocking. We have already said, in discussing this same table under Number, that the chief effects of the change from the method of the regular experiments to the present one, ought to be a lessening of the influence of the laws most dependent upon present peripheral excitation, and relatively to enhance the influence of those most dependent upon memory.

Law Four is of the former class; its influence, therefore, should be lessened in proportion as the stimulating influence of each pin is lessened. Comparing Block 92 of the new Table 5, with the corresponding figures of Block 60 of the "regular" Table 3 for the same region

—the forearm—we see that this actually happened. In Block 92 there is but little shortening of the judgments, with increased number of pins. The proper averages now read + 20.4, + 26.3, + 25.6, + 19.2 (indicating an irregular and slight shortening from left to right with increase of pins) as against formerly: + 17.7, + 13.3, + 9.1, + 7.6 (indicating a marked and regular shortening).

Law Three is of the class most exclusively based in memory processes; its effect by the new method, therefore, should be enhanced, as plainly it is. The average error is markedly greater throughout, and the general drift throws the increased error more constantly toward over-estimation. This shows so plainly in the tables that the figures need not be repeated here. A striking item of confirmation of our interpretation of the compensating influences of the several laws, is shown in the lower line of Block 89, where, lacking the compensating influence of Law Four as described in discussing the corresponding lines of Table 4 on page 103, the number of correct judgments no longer read decreasingly from left to right, but increasingly, as lacking the influence of Law Four they ought to read (38, 41, 40, 40, in Block 89, and 57, 40, 49, 48, in Block 60).

Law Two being little affected by the new method, holds its course even more obviously than it did in the regular experiments; and this, because lacking in a

greater degree the disturbing influence of Law Four, is what properly it should do.

§ 29. Table 7 furnishes other peculiar evidence for our laws. It records the results of practice. We should suspect *a priori* that the effects of practice would not improve all our laws of judgment equally. The law which we should most expect to be improved by the specific practice of these particular experiments is Law Two; this would chiefly result in heightening our familiarity with the precise regions worked on; which in turn would enable us to disassociate more sharply and distinctly the local percepts around each pin. Particularly would this apply to our percepts of the end pins, for the reason that our attention in judging distance is proportionally more bestowed upon these than upon the intermediate pins. Consequently, since improvement with reference to the end pins would mean, on the whole, improvement with reference to the longer distances, we should expect that the chief consequences of practice would be increased accuracy in judging the longer distances, and more pronounced effect of Law Two throughout. Examination of Blocks 110 and 113 (Table 7) discovers this to be just what took place; the average error of the longest distance is now —5.4 as against —12.8 formerly for the same region—the forearm—in Blocks 55 and 60, Table 3; and the average number of correct judgments increases now for the dis-

tances 1 to 3 cm. by the series of figures 29.5, 33.0, 39.5, 37.0, 78.0, as against the former series of 35.7, 44.0, 47.2, 40.2, 48.5.

The specific practice with the few distances actually used, would have little direct influence upon the memory habits of the other distances. Consequently the pull of these "possible habits" upon the drift of uncertainty, where there yet remained uncertainty, ought to be about the same as before. The figures confirm this. According to what we have said just above about the improvement in Law Two, we should expect to find the shorter distances remaining unimproved as compared with the longer distances. On the average, their judgments remain equally uncertain with their previous ones. The average error for 1 cm. was formerly + 52.4, and after practice was + 54.0. (Separate experiments we must not expect to agree wholly.) On the whole, therefore, these figures confirm what we should have expected as the behavior of Law Two, both as to average amount of error and the direction of its drift.

The matter is again confirmed when we look at the new effects of Law Four. It was noted as we became more expert in judging the distances, that we more and more based our judgments directly on the impressions of the end pins; there was less "reckoning" along from pin to pin, such as is based upon rocking. In other words, the effects of the intermediate pins entered less

and less into the judgment, and this is the same as saying that Law Four would have less effect than before practice. The obvious consequence of this ought to be, in the tables, that the judgments should exhibit less shortening than formerly from left to right through the columns with the increasing number of pins, and particularly this should be most manifest where there remained the greatest uncertainty. We may now observe that this is precisely what did happen. Not only do the footings of Block 113 show little of this shortening as compared with the footings of Block 60 (+18.9, +19.2, +12.2, +14.8 now, and +17.7, +13.3, +9.1, +7.6 formerly), but the top lines of the two blocks show no shortening in the 1-cm. judgments after practice, and marked shortening before practice (+69.1, +56.0, +44.3, +40.0 before, +48.0, +60.0, +55.0, +53.0 after).

Similar fulfillment of lawful expectations can be easily traced in the number-judgments of the same special experiment. The whole of Table 7, therefore, again affords striking confirmation of our thesis in general.

§ 30. A word must be said of the special experiment reported in Tables 8 and 9. If our general discussion, and in particular that part referring to Law Four, is correct, then we ought to expect that straight-edges or full lines of cardboard, pressed upon the skin, should awaken more accurate judgments than our lines of pins.

It ought to be a case of Law Four with the number of pins raised to infinity. Tables 8 and 9 report an experiment for testing this matter. In considering the results it should be borne in mind that the pins give much sharper impressions than do the card edges. Yet, notwithstanding that fact, the experiment is an interesting confirmation of our general doctrines. The average errors for the scale of distances on the forearm, Block 60, in the regular experiments ran as follows: + 52.4, + 27.1, — .1, — 6.9, — 12.8, while the corresponding errors with the card edges, Table 8, were: + 31.4, + 8.9, + 2.7, — .6, — 7.8. Correspondingly for the number of correct judgments we have for the pins: 35.7, 44, 47.2, 40.2, 48.5; and for the cards: 53, 50, 60, 72, 83. The total averages are: "pins," + 11.9 and 43.16, as against "cards," + 6.9 and 63.8. Table 9, where the cards were pressed without rocking, also yields its share of confirmatory evidence of Law Four, and both Table 8 and Table 9 are full of points confirmatory of our other laws, but we have not the space here to consider them.

§ 31. A glance at the summaries of Experiment A, exhibited in Table 10, shows again the integrity of our laws in a strikingly impressive manner, proportionate to the extensive field of confirmatory experimentation grouped into a single view. But for the present we must leave our special subject of distance to study higher spatial complications.

NUMBER-JUDGMENTS BASED ON TWO DIMENSIONS.

§ 32. In the Experiments B, C and D our investigations attack the psychology of two-dimensioned space. Necessarily we shall make but little headway with it; our "heaps of figures" will show rather what in the future is to be done, than reach complete demonstration of any kind.

Naturally we should first inquire in this new domain, whether the laws of number and distance already discovered are carried over into the formation of the new and more complicated judgments.

Number.

§ 33. We will first follow the laws of number. For these we must study Tables 11 to 17. They relate to Experiment B, which was conducted with pins arranged in triangles and squares, the distance categories remaining as before.[1] [The experiments subsequent to B do not involve number-judgments.]

[1] About these B tables in general, a preliminary word of caution is needed. By a great fault not appreciated in laying out the experiment, the highest category of pins both with the triangles and with the squares was not carried down through the shortest distances. As a consequence there is much complication in the operations of Law One. This is illustrated by any one of the blocks, for instance Block 196, Table 14. It will be noted there,

That Laws Two and Three both hold good throughout this new set of number-judgments, is seen by slight examination, but certain peculiarities, to be observed here and there in the wider course of general integrity, demand closer consideration. Perhaps the thing that first strikes us is the fact that the number of correct judgments, and particularly in the short distances, is on the average much greater than when the pins were arranged in a single line. What has our hypothesis to say of this? Suppose three pins, A, B and C, to be set in the corners of an equilateral triangle of 1-cm. base. According to our foregoing discussions, our ability to perceive these to be "three" will depend upon the predominance, in the combination of these three particular points of skin during life's experiences, of "three termed successions" above all other modes of coördinately stimulating them. What we must now ask is whether, other things being equal, this mode of combination is more likely to occur when three points are arranged in a triangle, than when in a straight line. Of course it is difficult to know when the "other things"

that the correct judgments in the "VI-pin" column fall off greatly in the larger distances where the other category of "VII pins" has been introduced, from what they were above in the shorter distances. Plainly this is the effect of chance and from opening a new and higher category which may possibly be judged. The effects in the averages are also somewhat disturbed. With due care, however, the results may be used comparatively without falling into grave errors.

are sufficiently equal for just comparison; as, for instance, with reference to the distance between the pins. In the above supposed triangle the distances average 1 cm. apart. Perhaps the straight-line category coming nearest to this in Experiment A is that where the end pins are 1.5 cm. apart, and the average distance between the three pins is 1 cm., the same as in the triangle. Assuming these two arrangements as concrete examples for our comparison, we observe that the triangular offers greater likelihood for successive stimulation than does the lineal arrangement. This is apparent if we consider the possible movements, in the plane of the skin, of a right line which is to be considered with reference to its stimulations of any three given points in the skin. If the three points are in a straight line p, the moving line l must occupy some position with reference to p that can be determined by the angle x between the lines. For our problem the movements of l must be computed between the values for x of o and 90°. But with $x = o$, that is when l is parallel with p, all movements of l over the skin would never be able to stimulate the three points in any sort of succession. This state of things could never happen with the points arranged in any sort of triangle. It is easy, therefore, to show by calculation that, whether the stimulations be made by continuous movements tangentially across the skin, or by vertical pressure upon

the skin in successively varying positions of the stimulant, the chances for the proper successive combination requisite to the development of the threefold form of numerical perception for the three points would be much greater in life under triangular than under lineal arrangement. The advantage in favor of the triangular arrangement is markedly extended by the very important sort of genetic differentiation arrived at through the principle of Concomitant Variations.

§ 34. The empirical fact that our experiments show the numerical judgments to be more accurate under triangular than under lineal arrangement of the pins would, therefore, if clearly demonstrated, be in strict accord with the theoretical demands of our general thesis; and, having the theory more fully before us, we must now examine this demonstration in our tables more particularly theretoward.

Assuming that the "III-pin, 1.5-cm." results of Experiment A may be compared with the "III-pin, 1-cm." results of Experiment B — an assumption which, as we will presently show, favors the right-line arrangement — we get from the several tables the following averages for number of correct judgments and for average error. The figures in the left-hand column refer to the lineal, those in the right-hand to the triangular arrangement:

		Line.	Triangle.	
Tongue	Block 5 " 10	99 + .3	98 + .6	Block 128 " 133
Forehead	Block 25 " 30	27 + 24.8	26 + 49.3	Block 148 " 153
Forearm	Block 45 " 50	34 + 9.3	28 + 38.7	Block 173 " 178
Abdomen	Block 65 " 70	19 + 39.8	46 + 32.0	Block 196 " 199

The above figures are given without correction being made for Law One. To make such corrections we should have to subtract much larger amounts from the "triangle" results than from the "line" results; we should have to do this because of the difference in position relative to Law One of the "III-pin" column in the two cases. I will not attempt the proper corrections, but it will be evident to any one upon due consideration that, within very safe estimates, they would demonstrate conclusively the greater accuracy of the judgments under the triangular arrangement than under the lineal.

Upon the same basis of average distances, if we compare the 3-cm. judgments of Experiment A with those at 2 cm. in Experiment B, we get the following:

		LINE.	TRIANGLE.	
Tongue	Block 5 " 10	99 + .3	99 + .2	Block 128 " 133
Forehead	Block 25 " 30	60 + 4.3	56 + 29.8	Block 148 " 153
Forearm	Block 45 " 50	27 + .9	40 + 28.6	Block 173 " 178
Abdomen	Block 65 " 70	28 + 29.1	50 + 24.6	Block 196 " 199

These figures also, when proper corrections should be made for Law One, would show the superiority of the triangle judgments to be very marked.

§ 35. It should now be noted that the above method of averaging the distances is very unfair in favor of the lineal arrangement. The judgments at 2 cm. are better than at 1 cm., and manifestly it would not be right to average 100 of the former against 200 of the latter. But this is practically what we do in the above method when we make the 2 cm. of distance between the two end pins in the lineal arrangement offset two distances of 1 cm. each in the triangular arrangement. This should be borne in mind in making corrections in the two above tables and for the comparisons now in hand all through.

§ 36. So much for three pins. A similar comparison with the foregoing may now be made for the "four-pin" judgments. For this, by the above method of simply averaging the inter-distances, we should have to compare the 1-cm. judgments of four pins in a square, with the 2.5-cm. judgments of four pins in a line, and such violent errors would arise from offsetting 2.5-cm. judgments with two and a half times as many 1-cm. judgments as to make such comparisons wholly inadmissible. We can, however, arrive at the desired information by a more satisfactory method. In Table 17, Block 216, we find the total average of correct judgments summarized for all the regions of skin worked on, to be: for "III pins," 48.95, and for "IV pins," 54.60. In Block 217, the corresponding average error for "III pins" is + 23.2; for "IV pins," + 17.0. As the conditions of Law One were precisely similar for these two sets of figures, they show conclusively that four pins in a square are more accurately judged than three pins in a triangle. We have already demonstrated in our experiments that three pins in a triangle are judged better than three in a line. To complete our proof, therefore, that four in a square are judged more accurately than four in a line, we have but to show that three in a line are better judged than four in a line. It is true that the general summary of the line experiments, Table 10, Block 118, shows 49.2 correct judg-

ments for "IV pins," and only 37.7 for "III pins." But before we take this as evidence that four pins are better judged than three, we must again make the proper corrections. We must remember that all through Experiment A we found over-estimation; that the position of the "IV-pin" column under Law One apparently offset this general over-estimation, making the "IV-pin" judgments appear unduly accurate, while the position of the "III-pin" column augmented the over-estimation, and heightened the errors. The figures as given, therefore, show an illusive comparison. The illusion is greatest where there is the greatest over-estimation, namely, in the shorter distances. It will be observed that the longer distances throughout give undoubted evidence, even without proper corrections, that three-pins are fundamentally judged with greater accuracy than four, the arrangements being the same. But making the proper allowances for the compensations between Law One and Law Two, and the fact which common sense would assert from the outset, that in similar lineal arrangement three pins are judged more easily than four, will, I think, receive from the figures of Experiment A throughout most unmistakable demonstration.

This being so, it is therewith also demonstrated that four pins in a square are judged better than four pins in a line, which was the main proposition under consideration.

§ 37. In the above paragraph I have incidentally stated the important fact brought out by Experiment B, that four pins in a square are judged better than three in an equilateral triangle of the same base. Why this should be so under our hypothesis, while "common sense" would expect the contrary, I have space here to demonstrate only partially. We may anticipate that we have to do here with matters of experience conditioned by geometric arrangements. By Law Two, pairs of points are increasingly disassociated proportionally to their distance apart. The diagonal points of the square are further apart than any pair of points in the triangle. By the mere law of distance-average, therefore, the square should rank above the triangle. No doubt the difference between the lengths of the hypotenuse and the sides of the square give a favorable "cue" in reasoning out that the figure pressed on the skin must be a square and therefore has four pins. But we may note as to this, that while the difference of length between the inter-distances in that category of our experiment where four pins are arranged in a triangle —one being in the center—is greater than in the four-pin square, yet the judgments for such an arrangement of four pins are inferior to those of the square of the same base. (Block 216, IV pins in triangle 47.20, in square 54.60.) We must consider, therefore, that the superiority of the square over the triangle in numerical

judgment, is fundamentally rooted in the integrity of Law Two carried over into the more complicated judgments of two-dimensioned space. In this light the whole matter becomes confirmatory of our genetic hypothesis, and highly instructive as to the intimate formation of those mental processes which are a degree more complex than the most simple ones.

§ 38. As bearing on the above I have only room to note in Table 15, where no rocking was permitted, that the IV-pin square still shows superior to the III-pin triangle, while in Table 16, where the mind was forced to neglect the geometric impressions and to attend alone to the points actually felt, the averages show the III pins to be judged correctly 9.2 times in the triangle, and the IV pins only 8.6 times in the square. That is, when the geometric influence is shut out, the judgments fall back to the common principle that three impressions may be better distinguished than four.

§ 39. Turning from Law Two to Law Three we discover upon slight study that, within categories as nearly similar as could be chosen for comparison of the different arranging of the pins, the same general drift of over-estimation is manifested throughout Experiment B as we discovered throughout Experiment A; also, over-estimation is greatest now in the same relative places as formerly, namely, in the shorter distances. These above facts are, perhaps, evidence for laws

already redundantly confirmed, but the subject gains extended interest when we examine, from another point of view, the amount and the distribution of the errors in the B experiments.

We could have expected from Laws One and Three, that the greater the number of numerical categories used in any experiment, the greater would be both the amount and the drift of the errors made under these two laws. For instance, if we made new investigations like those of Experiment A, but with numbers of pins ranging from III to IX (as in Experiment B), in place of from II to V, as formerly, we should expect the amount and the range of errors to be much increased. Upon the face of it, the possibility of error where the judgments may range from III to IX is greater than when they are limited between II and V; both the mathematical chances are greater under Law One, and the psychological chances are greater under Law Three. When, however, we study the results of Experiment B, we find quite the reverse of what, from the above, was to have been expected.

By way of examining into this we must first grasp more clearly the relative amounts of error made in the two experiments. We had trouble in getting an exact basis for this comparison, but in rough ways we may yet get truer ideas of it. All the conditions, save those whose results we are seeking to measure, favor

the II-pin judgments; that is, other things equal, II pins ought to be judged better than III. If, now, the new triangle judgments happen to exhibit a *less* amount of error than did the old II-pin judgments, we may then justly take the amount of this improvement to be a partial measure of the new conditions of the experiment. For a full comparison one must go to the full tables, but we will bring forward a few test items. In the following table we will compare both the *maximum* amounts of error made for II pins in Experiment A with those for II pins in Experiment B, and also the corresponding *average* errors:

	Maximum Error.		Average Error.	
	II Pins.	Triangle.	II Pins.	Triangle.
Tongue	0	+ .6	0	+ .2
Forehead	+ 84.2	+ 69.1	+ 35.0	+ 27.6
Forearm	+ 70.0	+ 58.2	+ 45.4	+ 31.7
Abdomen	+ 109.9[1]	+ 32.0	+ 65.5[2]	+ 33.9

Similarly, the total average in the general summaries (Tables 10 and 17) are for II pins +41.6, and for the III-pin triangle +23.2. For a still rougher comparison, the grand averages of the same summaries give us the

[1] See upper left-hand corner of Block 70, Table 4, and correspondingly for other tables.

[2] See footing of left-hand column, Block 70, Table 4, and correspondingly for other tables.

following errors: Experiment A, + 5.8; Experiment B, +2.1 (lower left-hand corner, Blocks 121 and 217). The superiority of the triangle arrangement is obvious everywhere without comment.

§ 40. The above figures having furnished us a clearer demonstration of the *amount* of superiority of two-dimensioned judgments over the lineal, we must look at certain differences in the *distribution* of these errors under the two experiments. We get at these quickest by an illustration. If we turn back to our old typical Block 65 and read the top line, we get the correct judgments at 1-cm. distance for II, III, IV and V pins, respectively, as follows: 7, 32, 59, 55. We remember the explanation of this remarkable increase of apparent accuracy from left to right; that it was due to the drift of the great uncertainty of the short distance-judgments. If now we turn to the top line of the corresponding Block 196 of Table 14, we get, with a similar ascending series of pins, a descending series of correct judgments, as follows: For triangles 46, 32, 31; and for squares 44, 19, 28. We have no longer the remarkable increase from left to right due to drift of error under Laws One and Three. What is the trouble? Are these laws inoperative here? No! but we have, in a particular but legitimate exception, a remarkable proof of the general integrity of these laws everywhere, both throughout Experiment A and through-

out Experiment B. We have recalled that the drift from left to right in Block 65 was a drift of *uncertainty*. If now we look at the top line of Block 199 we see that the uncertainty there is very small as compared with the corresponding line of Block 70. The maximum error is for the former +32.0; for the latter +109.9. The average error for the former is —.3; for the latter +35.7. The drift of error from left to right in the new experiment ought, therefore, to be proportional to this marked decrease in error. And so it is. Apparently there is none whatever, and the number of correct judgments actually decrease from left to right. It is possible that corrections for Law One would still leave a small drift under Law Three, but in any case it would be so small as to constitute a peculiar proof of the integrity of this law carried up into the more complicated judgments of two-dimensioned space. Of course this is but a sample of proof which would be augmented should we extend our examinations. For the forehead and for the forearm the amount of error is less in Experiment B than in A, but not so much less proportionally, as in the above sample taken from the abdomen; accordingly, the drift under Law Three should be less than formerly, but not so much so as in our above illustration. This is just what occurred, as the following figures taken from the tables will demonstrate. They give the top lines respectively of Blocks 25, 30, 45, 50, 148, 153, 173 and 178 :

Experiment A.

FOREHEAD.

5	16	61	64
+ 84.2	+ 43.5	+ 1.5	− 8.7

FOREARM.

26	31	45	23
+ 70.0	+ 18.1	− 11.7	− 27.3

Experiment B.

FOREHEAD.

26	28	54	27	54	78
+ 69.1	+ 35.0	+ 12.9	+ 53.5	+ 28.9	− 8.9

FOREARM.

28	28	34	34	35	44
+ 58.2	+ 21.4	− 9.5	+ 32.4	+ 10.3	− 21.5

Here, for the forehead in Experiment A, between II and V pins, we see a drift of correct judgments from 5 to 64, and of amount of error from +84.2 to −8.7; while in Experiment B the corresponding drift between III and IX pins is only 26 to 78, and +69.1 to −8.9; and so similarly for the forearm.

§ 41. Had we space, we ought to consider further the distributing influences of our laws under the different conditions of our experiments and for the different regions of the body; but this we must now leave to the individual student. Compelled now to pass on to other matters, we may summarize our imperfect study of the number-judgments in our new Experiments B, as fol-

lows: We observe in the new workings of our laws both certain modifications of old traits running parallel to definite changes in the conditions under which they act, and also entirely new traits due to new conditions. We find all these manifestations agreeing with each other and conforming to the reasonings of our general hypothesis; we must, therefore, admit them to be strong evidence for its truth.

DISTANCE-JUDGMENTS BASED ON TWO DIMENSIONS.

(*Experiments B, C and D.*)

§ 42. The elementary law of Association is that the resultant state at any moment is the indissoluble product of the sum of all the tendencies active at that moment. It is fundamental to all that I have heretofore said, that this law holds good for the stimulation of each and every possible combination of nerve-ends. When we stimulate a single nerve we get a specific effect which expresses the tendencies developed for that definite stimulation. When two given nerves are stimulated we get a different effect, also specific, and expressing the tendencies developed coördinately for the given combination as stimulated. Just what relationship the specific state (which expresses the sum of

combined stimulation) bears to the several specific states (which, respectively, on occasions express the stimulation of each nerve separately), our science does not now with confidence suggest. We cannot yet formulate the coördinate tendency, in terms of the several separate tendencies. But while we may not determine its particulars, we may demonstrate that there is such a relationship; and this is to be my present thesis. Stated more explicitly it is that: The actual effect of any combination of nerves is partially the resultant of the combined experiences of the given combination, and partially, also, is to be traced back to the experiences which have influenced and developed each element or possible sub-combination of elements in the total combination separately. It is the latter part of this statement which, in approaching more complicated distance-judgments, we must especially consider.[1]

§ 43. We may profitably bring this matter before us by considering one of the triangles of Experiment C. The sides of these triangles were formed by straight edges of cardboard, the card being folded as when forming the sides of a paper box. We are to study the coördinated result of pressing one of these lineal figures upon the skin. Call the triangle *ABC*. Now, by our

[1] It must be borne in mind that we never suggest that the *resultant mental state* is other than an indecomposable specific whole.

thesis, upon the simultaneous stimulation of the lines A, B, C, there will *tend* to rise every effect common to the separate stimulation of every sub-combination of nerve-ends possible under the laws of permutation and combination for the total number of nerve-ends in the whole lineal triangle. We do not say that the actual effect will be the resultant *solely* of the sum of these tendencies. On the contrary, every time the whole triangle is affected, as in our present case, there will be left thereby a direct modification of the habit of reaction of the total combination, and this modification will, in some degree, manifest itself in all subsequent activities of the total combination. But we are now to consider the tendencies of the many possible sub-combinations.

If we examine the case of any two points, y and x, in the perimeter of the triangle (one, at least, of the same not being a corner) we see, by our general thesis, that the distance-judgments based upon the separate stimulations of any such points would be shorter than that of the sides of the triangle. Consequently, by our present thesis, the influence of all such tendencies, in the sum of tendencies resultant from stimulating the whole triangle coördinately, ought to shorten any distance-judgment simultaneously based thereupon. If this resultant judgment is to estimate the length of the sides of the triangle, or of one particular side, then,

although the "developed tendency" corresponding to the side may be the chief factor in the formation of the judgment, yet these other tendencies, being in activity by reason of actual peripheral stimulation, they cannot be wholly got rid of and will enter into the sum total of present influence; and, being shorter than the tendency corresponding to the full side, they will shorten the resultant judgment from what it would be, were the same distance measured simply between two pins. If, thus, our thesis is correct, all the distance-judgments for triangles in Experiment C ought to be shorter than judgments of corresponding distances in Experiment A.

§ 44. Before we examine our tables to test this theoretical conclusion, a relative matter must be considered. It concerns the "sharpness" of the different modes of stimulation. Up to the present, to avoid confusion, I have excluded the factor of "pure sensibility" from our studies. In proper place, I shall bring in special investigation to demonstrate what I shall state here dogmatically, namely, that, within certain limits, "*sharpness of stimulation*" shortens the resultant distance-judgment. Now, according to this, and since the pin-points of Experiment A furnish a much sharper mode of stimulation than the card-edges of Experiment C, in comparing the judgments of the two we must bear in mind that those in Experiment A are shorter than, for a perfectly just comparison, they should be. If in

the actual results the theoretical demands are sometimes apparently unfulfilled, due allowance must be made; and if they are fulfilled without such allowance, the proof of our thesis must be considered as the more marked.

§ 45. Toward making the proper comparisons, I first present the following table. In the vertical columns are given alternately, the maximum and the average over-estimation taken correspondingly from the "*average*" distance-judgments of Experiment A, and the triangle distance-judgments of Experiment C. I have used the "average" here in place of the II-pin judgments referred to in the theoretical discussion, as it may be claimed that the latter are unduly lengthened by the distributing influence of Law One. The "averages," since they are free from such criticism, are in theory equally eligible to the proper comparison and, in fact, are shorter, as, by Law Four, they should be, and will, therefore, if they stand the test, demonstrate our point in hand even more profitably than would the "II-pin" judgments:

	Experiment A.		Experiment C.	
	Reference Numbers.	Over-Estimation.	Over-Estimation.	Reference Numbers.
Forehead	Table 2, Block 35	+ 19.2 + .2	+ 7.0 − 1.3	Table 18, Block 229.
Forearm	Table 3, Block 60	+ 52.4 + 11.9	+ 40.0 + 5.3	Table 19, Block 247.
Abdomen	Table 4, Block 80	+ 26.0 + 3.5	+ 43.0 + 4.9	Table 20, Block 265.
Forearm, Method (a)	Table 5, Block 92	+ 98.7 + 11.9	+ 41.0 + 2.4	Table 21, Block 283.

These figures, with the exception of those for the abdomen, confirm our theory, even without allowance being made for sharpness. And as the abdomen, owing to the thinness and tenderness of the skin, is just the region where sharpness of the impressions from the pins would be of greatest effect, as compared with the rather dull feeling from the paper triangles, I think the evidence of the table is beyond question. Particularly we call attention to the figures for the last line of the table; these are for the "(*a*) method," where the apparatus is applied evenly, without rocking. There is reason to believe that the effects under present discussion would come out purest in this method. It is, therefore, of interest to note that the contrast of the two sorts of judgments is more marked in these results than anywhere else in the table.

§ 46. The straight-edge judgments of Experiment A (see Tables 8 and 9, discussed on page 109) were shorter than those for the same distances measured by pins in straight line. Theoretically, for the reasons already given, our triangle-judgments should be shorter than either. Comparing the average errors, respectively, from Tables 8 and 9, with the corresponding figures for triangles in Tables 19 and 21, we get as follows:

	Experiment A.	Experiment C.
	STRAIGHT-EDGE.	TRIANGLE.
Regular Method....	+ 6.9	+ 5.3
Method (*a*).............	+ 8.9	+ 2.4

Again, without allowance for sharpness, the average errors all show the superiority of the triangle-judgments, and again the contrast is most pronounced in the purer (*a*) method. But allowance for sharpness should be made even here. True, the straight-edges were made of paper, as well as the triangles. But, with the former, where the hard cardboard is cut squarely off, the corners are nearly as sharp as pins, whereas the same cards, when fitted together as perfectly as possible into the obtuser figures of Experiment C, make corners that are far duller and more indistinct. On the whole, therefore, these figures speak pronouncedly the superiority of the triangle judgments over even those of single lines.

§ 47. But we now come to evidence, in bulk and unmistakable clearness confirmatory of our theories, outweighing all that we have previously reached. For

reasons so similar to those above given, for the superiority of the triangle-judgments over those for single lines, that they may be here omitted in detail, it is demanded by our hypothesis, other things being equal, that the sides of squares should be judged to be longer than the sides of triangles of equal length. Moreover, the diameter of circles should be judged to be less than the sides of squares, and to be greater than the sides of triangles, the real distances all being equal. Experiment C was specially designed to investigate these matters, and the results are among the most conclusive which I have to present. Throughout the whole set of C experiments, both the figures for number of Correct Judgments and those for Amount of Error, balancing themselves within small margins of variation, demonstrate, in accordance with our thesis, the proper relationships of accuracy and foreshortening, respectively, as between triangles, circles and squares, in a way that is truly remarkable; is the more remarkable since the conditions are here perfect for comparison throughout—no allowances or corrections having to be made for anything. We need not repeat here any figures for the results, for they are everywhere plainly to be understood, upon reference to any of the Tables 18a to 22a, inclusive. The triangles are always judged shorter than the circles, and the circles shorter again than the squares. The importance of such a mass of testimony,

however, must not be underrated because of the brevity in which it can be mentioned.[1]

§ 48. But important as this mass of evidence is, we find it duplicated in the several tables of Experiment D (18b to 22b, inclusive). Throughout these, we again find the sides of triangles judged to be shorter than the diameters of circles, and the latter to be shorter than the squares; for all of which, as before, we refer directly to the tables.[2]

§ 49. Experiment B furnishes another mass of evidence. Here we have no circles, but the arrangement of four different number categories, similarly in triangles and in squares, gives opportunity for instructive variations. I will first ask you to compare the averages of

[1] In studying the tables for C and D the figures for the *number of correct judgments* must be interpreted with care. For instance, these respectively for triangles, squares and circles, read in Block 226, for the top line 86, 32, 72; and for the bottom line 48, 88, 62; yet both are precisely what they should be to demonstrate our thesis throughout. It must be recalled that Law One operates here as elsewhere. If we glance at Block 229, showing the corresponding amounts of error for the above judgments, we see the explanation of them at once: In the top line there is undue over-estimation throughout (Law One); consequently the relatively shorter judgments of the triangles are more frequently right than the squares or circles; hence the figures 86, 32, 72. In the bottom line there is undue foreshortening throughout (Law One); consequently the triangles here are less frequently right; and hence the figures 48, 88, 62. The relative length of triangles, squares and circles is perfect everywhere.

[2] It should be recalled here that for the D apparatus the figures are made of cork and present a smooth, even, solid surface to be pressed upon the skin.

the whole four categories for the triangles, with the corresponding averages for the squares. I need not reproduce the figures, but examination of Tables 11 to 17, inclusive,[1] shows, with scarcely an exception, that the average judgments of triangles (in column headed "T") are shorter than those for squares (column headed "S").

Next I will ask you to compare the distance-judgments of similar figures which contain different numbers of pins. According to our doctrine of Average Distances, it should hold that, of two triangles of equal length of sides, one arranged with III pins (a pin in each corner) and the other with IV pins (one pin being in the middle), the judgments of the length of the sides of the latter should be shorter than those of the former. In the second arrangement the influence of the three short distances, from the middle pin to the corner pins, would be brought into action. Accordingly, in the blocks of Experiment B, with the increase in the number of pins, should go shorter judgments from left to right through each of the four categories for triangles and for squares. The "IV-pin" triangles should seem shorter than the "III-pin" ones; those of "VI pins" still shorter; and "VII pins" shorter yet. So similarly for the IV, V, VIII and IX-pin squares. Examining, to test this matter, with scarcely an exception, throughout

[1] Blocks 143, 163, 188, 205, 212, 222.

the regular Tables 11, 12, 13 and 14[1] will be found precisely the results that our above discussion demands. The test is seen best, of course, in the footings — in their constant diminution through the first four columns, and again through the second four columns.

If we find the above rule less observed in the irregular "(*a*) method (Table 15, Block 212), the reason is not far off. By Law Three, the closer the pins the less would be the disassociation, and the greater the consequent over-estimation from uncertainty." This would work in opposition to the fundamental tendencies under discussion. In very short distances, with triangles and squares, putting in extra pins might work to lengthen the distance-judgments, just as we saw it do in the uncertain categories of Experiment A. Such a result would be most likely to occur with the method that gave the most uncertain impressions; in our case with the (*a*) method. Examination of the 1-cm. judgments of Block 212 will show this actually to have happened. Thus, while the bulk of the "regular" B experiments confirm the law of average tendencies, the apparent exceptions in Table 15, when understood, make the confirmation all the more striking.

§ 51. Were the principle of Average Distances alone considered, the sides of a "III-pin" triangle in Experiment B, should seem precisely equal in length to the

[1] Blocks 143, 163, 188, 205.

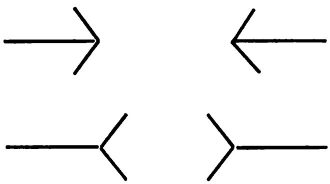
same distance measured between II pins, as in Experiment A, and the sides of the IV and V-pin squares in B should seem longer than corresponding distances in A. As a fact, in our experiments, all the judgments, both for triangles and for squares, are shorter in B than for corresponding distances in A. For this there are several interesting reasons.

When, in Experiment B, we press the "III pins" upon the skin, we *think* of a triangle, and not alone of three points; and to think of a triangle is to think of its lineal sides. In judging and measuring the pin-triangles, therefore, the *thoughts* are much of the same nature as when judging the lineal figures of Experiment C, and it may be partially for the same reasons according to which these last were judged shorter than equal distances in Experiment A, that the triangles and squares of B also seem shorter than the A distances.

It may be, also, that the simultaneous repetition of the same distance in the three sides of the triangle, or in the four sides of the square, strengthens the specific right-line tendency, and thus adds foreshortening opposition to the forces of uncertainty, which, as we have found abundantly demonstrated, drift toward overestimation.

But the chief factor in foreshortening our judgments of triangles is to be found, I suspect, in the tendencies of our attention. When we visually measure a triangle

we direct our attention particularly to some one side. Yet we do not wholly shut out of vision the two other sides. The presence of these in active stimulation continually tends to pull the focus of attention towards themselves. This throws into the "sum of tendencies" a lot of influence corresponding to what would actually happen, did the eyes sweep narrowingly down the two lines towards the apex, namely, a lot of shortening tendencies. It is easily to be understood, under our hypothesis, how this affects the resultant visual-judgment. The illusions discussed by Zollner, Brentano and others, of which the following cut is a familiar example, I should explain as allied phenomena:



To the extent that one visualizes in estimating dermal triangles, will his judgments be shortened in the above-described manner. If one does not visualize, he brings in some other auxiliary mode of conceiving a triangle, involving some similar principle. Perhaps, for instance, that of drawing, respectively, the thumb and finger simultaneously down the sides of a triangle to the apex, and measuring the parallel distances between them, in

terms of the muscular sensations of gradually closing the fingers. In any case there is a narrowing of the distance-element in the thought processes, which makes itself felt even when we use these processes in an auxiliary manner in judging dermal triangles. There are abundant reasons, under our hypothesis, therefore, why both the triangles and the squares of Experiment B seem shorter than the lines of Experiment A.

§ 52. Finally for this part of our paper, having made our general analysis of the formation of our distance-judgments from triangles, squares and circles, I think we cannot better fix the conclusions which we have reached, than by a brief examination of our old Laws Two and Three within these complex formations. Without exception, throughout the whole set of tables for Experiments B, C and D, inclusive, correction of Law One leaves an Average Error with a + sign. This demonstrates that the same over-estimation which occurred in Experiment A, holds over and manifests itself in the more complicated "two-dimensioned" judgments, made of similar distances, and from the same regions of the body as formerly. Moreover, the greater + errors are shown in the shorter distance categories. This demonstrates that the principle, according to which the over-estimation is distributed in simple judgments, is carried over to express itself with integrity in those complicated judgments which are largely

founded upon the simpler ones, namely, the principle that the greatest over-estimation occurs where there is the greatest uncertainty and by reason of the uncertainty.[1] The integrity of Law Three in these higher judgments is, therefore, demonstrated throughout.

Again throughout all these tables, without exception, the Average Errors of the longer distances are minus. This shows that the over-estimation, due to uncertainty, diminishes, and that the accuracy of the judgments increases throughout as the distances lengthen. Thus is demonstrated that the influences of Law Two also are carried over to act in the same manner and with the same integrity in the complicated judgments, whose formation they participate in, as they did in the elementary judgments, where we first discussed them.

§ 53. In taking leave of these subjects of Number and Distance in this section of our work, we may, therefore, summarize as follows: In our simpler experiments we observed certain principles and laws, which

[1] Marked confirmation of Law Three in the more complicated judgments may be noted by comparing the results of Experiment D with those of Experiment C. The apparatus of D is duller than that of C. The impressions are more uncertain. From this uncertainty rises greater over-estimation, which manifests itself proportionally throughout all the distances of D. Were the apparatus of the two experiments equal in sharpness, we should probably find the judgments of solid figures shorter than of lineal figures. It would be interesting to try figures formed of pins set solidly together.

we traced out undeviatingly in our more complicated experiments. The whole mass of phenomena we find conforming to, and to that degree explained by a comparatively simple, general hypothesis. We feel compelled, therefore, to admit this series of investigations as peculiarly abundant and convincing evidence in favor of the truth of this hypothesis.

JUDGMENTS OF FIGURE.

§ 54. All the "tendencies" of Number and all those of Distance are likely to enter into the construction of figure-judgments, and perhaps many more. That is, we *can* get a cue for the judgment that a certain figure in Experiment B is a triangle, from the fact that we distinctly feel *three* pins and only three. Or, we can suspect a lineal figure in Experiment C to be a square, from a sense of "*distance*" fullness "all the way round" the perimeter of the otherwise indistinguishable impression—a specific feeling which at once, to the expert, distinguishes a square from a triangle or from a circle. Or, we may reason out that one of the Experiment D figures is a circle from the fact that no corner is felt in the impression. By reason of these several modes of getting at the shape of any conventional figure, we are usually better able to tell that a triangle, square, or circle is a triangle, square, or circle than to estimate

its dimension, or count the exact number of its pins or corners. In conformity with all this, it is now to be noted that, throughout all the Tables 11 to 23, the number of correct judgments of figures everywhere exceeds the number of correct judgments, either of number or of distance.

By comparing the distribution, however, of the "figure-judgments" with that of the number-judgments in Experiment B, and with that of the distance-judgments in Experiments B, C and D, they will all appear so similar, at a glance, as to convince any one that the same laws which operated in the tables which we have already studied, are also felt in these figure-judgments. The general integrity of our old Laws Two, Three and Four, within this new field of mental processes, we may, therefore, accept without further comment.

§ 55. Table 23 demands a few words of special comment. It is made up from the averages of Tables 18 to 20, and shows the total averages of correct judgments, per 100, for each sort of figure and for each category of distance; and, also, it gives an account of the remaining *incorrect* judgments of each 100, in order to show what sort of errors were made, and how many of each. By this, we see that, in the shorter and more uncertain distance-categories, triangles are more correctly identified than circles, and circles than squares; the respective averages, for 1 cm., being:

	T.	C.	S.
Experiment C	65	44.5	40.5
Experiment D	57.5	45.5	38.5

And, in the longer and more certain distances, we see these relations almost precisely reversed; the respective averages, for 3.5 cm., being:

	T.	C.	S.
Experiment C	39.0	58.0	74.0
Experiment D	54.5	83.0 [1]	75.0

These results bring out the fact that in the shorter, uncertain categories we judge the contour of the figure chiefly by the acuteness of its angles. The triangle having the sharpest angle, we judge it most correctly; the circle having no angle, we pick it out next easily; the square, lying in contour between the two, is more difficult to distinguish than either. The triangle and the square, both having angles, are more frequently mistaken, the one for the other; while the obtuse figure of the circle, being more like the square than like the triangle, the circle and square are the pair next most likely to be mistaken, one for the other. All of this

[1] Evidently too large. See preceding Block 319.

is not surprising, and is not instructive till we observe that the discrimination of the acuteness of an angle is something very kin to numerical discrimination, or that, at least, to judge of the acuteness of the angular impressions involves a very low development of the distance elements, such as are formulated under our Law Two.

§ 56. On the other hand, this fact becomes doubly interesting when, looking again at our results, we find reason to believe that, in the longer and more certain distance-categories, the distance elements yield the chief basis for the judgments: (1) By Law Two, the distance elements ought to be highly developed in the higher distance-categories. (2) By the Law of Average Distances, where the distance-tendencies are highly developed and active, there, other things being equal, squares should seem longer than circles, and circles than triangles. (3) In the higher distances of our tables, the sign of our errors is minus (—). Consequently, by (1), (2) and (3), in the higher distance-categories of our Table 23, our theories demand a greater number of correct judgments for squares than for circles, and for circles than for triangles. This the figures quoted, two paragraphs above, show actually to be the state of the case. Since theory and experiments agree, we conclude, therefore, that the "distance-elements" are the active elements in these judgments, and that their presence

here, and their absence in the lower distance-categories, accounts for the reversal of the relations which the number of correct judgments, respectively, of triangles, circles and squares bear to one another, and which we observe in the figures above quoted.

§ 57. One other matter of peculiar interest to our general subject is afforded by Table 23. Our triangles are of smaller area than our circles and squares. Smallness goes with dullness and uncertainty of impression. When experimenting, therefore, in the mind of the subject, the notion of "triangle" soon gets intimately associated with the lack of clearness which is common to all the categories of figure in the shorter distances —triangles, circles and squares—all similarly. Another way of expressing the activity of Law Three in our figure-judgments is, therefore, by saying that, on the whole, the errors of uncertainty "run to triangles." There is greater uncertainty in the shorter distances than in the longer ones; and the number, respectively, of squares and circles that "run to triangles," in the 1-cm. judgments, as against those in the 3.5-cm. judgments, are: 41.0 and 24.5, against 12.5 and 5.0, for Experiment C; 45.5 and 24.5, against 19.0 and 5.0, for Experiment D.

Another expression of the same principle is discovered by comparing Experiment D with Experiment C. The Experiment D impressions being the duller (cork), more

judgments should "run to triangles" here than in Experiment C. Looking again at the figures quoted at the end of the above paragraph, or at Table 23, directly, we see it to be true, not only that the "run-to-triangle" errors are more numerous in the 1-cm. judgments than in the 3.5-cm. ones, but, also, they are more numerous in the D experiments than in the C experiments.

Throughout all our figure-judgments, therefore, we find the integrity of all our laws again confirmed, and the whole mass of empirical results standing in close and illuminating agreement with the several principles of our general genetic thesis.

THE MASS, INTENSITY AND TIME-ELEMENTS OF DISTANCE-JUDGMENTS.

(Experiment E—With a Moving Pencil.)

§ 58. The method and purpose of Experiment E have been explained on page 57. The general results may be briefly told. Upon all the regions of skin investigated, quick strokes seem shorter than slow ones, and light strokes than heavy ones. Quick-light strokes seem shortest of all, and slow-heavy ones longest of all. The precise results are given in Tables 24 to 27.

It would, perhaps, be a popular thing to explain these phenomena as matters of very simple association.

Other things being equal, short "distance series" occupy a short time, and long distances a long time. Thus we come, on the whole, to associate a short-timed movement with a short distance, and a long-timed movement with a long distance. Also, it is perhaps true that heavy bodies, on the whole, move both less far than light ones, and slower; and, thus, both directly and indirectly, heavy impressions become associated with the notion of "long distance" and light bodies with that of "short distance."

There is no doubt whatever that such associations can be and frequently are formed, and no doubt that, through *reasoning processes* based upon such associations, we frequently arrive at misconceptions. But I am not at all sure that such mis*perceptions*, such as our experiments deal with, are rightfully explained in this way. Such a theory, when closely examined, seems to demand first, a perception or sensation, having a nature in and of itself; then second, quite another associated idea, awakened *afterward* by this perception or sensation; and then a third, still more subsequent state or judgment, formed from the mutual reaction of the first and second described processes; and this seems to me a clumsy affair, involving both more dogmatic assumption and more psychological speculation than is necessary. Why not say that the first "whole" mental state that rises to any sort of definiteness, and which is the first and

native reaction to the outer impressions, as a whole, has itself, and at once, a specific nature, which chiefly explains our phenomena in question?

Of course, all perceptions involve memory processes more or less, and in turn we must think of these as involving lapse of time. But it is one thing to talk about Association of Ideas, and quite another to talk about Association of Tendencies. It must be clear that, according to our thesis from the first, in us *every* differentiated mental state, however simple, must be the specific correspondent of a specific Sum or Association of Tendencies. Even if we could isolate and stimulate any one single peripheral fibre, it is probable that the cerebral tract which would immediately react thereto, would be one that commonly reacted to more than that one fibre, and whose habits of reaction had been partly moulded and developed by the activities of other fibres. Its reaction, therefore, even to the single fibre, would, by our thesis, involve a "sum of tendencies" dependent upon fibres and forms of outer impressions other than the single one isolated by our proposition. If such "Sums" are what we are to mean by association, very good; but they are very different matter from the association of our text-books and lead to very different psychological explanations.

Asserting that a quick-light stimulation of a definite stretch of skin does give us a mental reaction which at

once or as soon as it culminates to any definiteness has a specific nature which *is* a perception of a definite short distance,[1] and similarly that a slow-heavy stimulation of the same stretch gives at once a perception of definite longer distance, we have now to inquire how our general thesis would account for these phenomena.

§ 59. Whatever cortical part it is that responds to peripheral stimulation, when it so responds expression is given to the result of two quite independent sources of influence — the one central and the other peripheral. It is of the very essence of our thesis that the central influences or tendencies of distance are serial tendencies. The peripheral influences of our experiments in hand are also serial. But what we may chiefly note is that the two influences are likely to tend and usually do tend to different series. The central influence is an already developed habit. The peripheral influence is whatever the experimenter makes it. Theoretically, if the peripheral influence acted alone, a specific mental result would rise in consciousness correspondent to and

[1] Of course, such a perception does not come to us all *named* and *recognized*, we will say, as "one centimetre" from the outset. That *is* done subsequently. Yet, it never would call up the proper recognition and name, did it not have a specific nature of its own to start with. And what I claim is, that this first-blush has a specific *distance* nature from the start, and of such complicated development, central and peripheral, that it should never be looked upon as a simple sensation, but as constituting the very fundamental nature of all distance perception.

expressive of a distance series of definite length. If the central influence acted alone, a result expressive of a distance series of different length would rise. Acting jointly, as under outward stimulation they always must, the two influences give rise to a distance perception expressive of a distance series mediate between the two theoretical ones.[1] That a quicker or shorter-timed peripheral movement mediates a perception of shorter distance, and a slower or longer-timed movement mediates a longer perception, is in such close harmony with our theory, as to the *origin* of our central distance tendencies in general, as to need no special discussion. The effects of quick-slow stimulation in Experiment E are seen, therefore, to accord with our general thesis, and to be explainable thereby.

§ 60. The effects of light-heavy stimulation are to be accounted for quite differently.[2] It has been the essence of our general thesis from the start, that *mental*

[1] I do not mean to tie the matter up to a hard-and-fast mathematical relation between the passing series and the average of all the series of our experience *alone.* Lots of other things, as nutrition, biologic growth, general health, above all, the nature of the total content of our mind at the moment we make such judgments, come in to modify, in some degree, the particular distance tendencies which chiefly dominate the focus of attention.

[2] Professor Wundt (Grundzüge d. Phys. Psy.$_3$, II. 19) states the fact that the same distances appear longer under heavy than under light movement, but gives no exact data, nor any further explanation of the phenomena than that they are dependent upon "many physiological and psychological conditions."

distance is but another expression for mental time-form. Not till we have time-form can we have "series," and in "series" is the origin of all mental distance. Right here we should note that mental processes may differ in other ways than in time-form. For instance, the most elementary feelings may differ in "mass"; the specific feeling resulting from the reaction of a large nervous mass is not the same as results from a smaller nervous mass. A big pain is not the same thing as a little pain, even outside of intensity. Now, just because mental processes may differ in other respects than in time-form, and because mental distance is solely time-form, therefore, we should clearly observe that, by our thesis, mental distance cannot be conditioned by *mental* mass. Definitely stated: The time-form remaining the same, "more" or "less" *feeling* can never constitute *mental* distance. I will emphasize this by an illustration. Our thesis has declared from the start that, other things being the same, originally, the *simultaneous* stimulation of any number of nerve-ends would, mentally, give the same "distance" result, whatever the peripheral arrangement of these nerves—whether in a bunch or in a straight line. Not until *successive* stimulation came in, could the *mental* results differ in "length." Yet, it is to be observed, the results of simultaneous stimulation could differ otherwise than in "length"; for the stimulation of many nerves would give *more* feeling than

would the stimulation of a few. Thus, there could be more *feeling*, yet not more "length"; more "mass," yet not more "distance."

To recognize this independence of mental "mass" and mental "distance" is very important, and failure to do so constitutes a crucial error in all psycho-physical experimentation into which "space" enters in any way. The root of this error lies in assuming, outright, that the *amount of stimulation* applied, peripherally, in any given unit of time, necessarily bears a direct proportion to the amount, or *"mass," of feeling* resultant therefrom in the corresponding unit of time. No doubt it is true of the physical activity to which the feeling *ultimately* corresponds, that the amount of *its* activity bears a direct proportion to the "mass" of the feeling. But the mistake comes in by assuming that the amount of *this ultimate* physical activity is always, for correspondent units of time, directly proportional to the amount of *peripherally applied stimulation.* As well might we assume that mental reaction-times always correspond directly with the time-form of the outer stimulation, and, therefore, are *independent* of the amount of peripheral area stimulated—(that the reaction-time to a small heated surface is the same as to a larger heated surface, the intensity of stimulation being a constant). Indeed, to assume, in Experiment E, that the resulting distance-series would be, in length, independent of the heft, or

"amount," of stimulation per time-unit of the moving pencil, would be identical with asserting that mental time-actions are independent of *amount* of peripheral stimulation. In view of our general thesis, and of all it has explained to us in our foregoing experiments, until it can be shown that the distance-series, or tendencies, do not change in length, proportionally to the heft of the pencil, I am inclined to hold to the doctrine that the differences shown between the judgments of the "light" and the "heavy" categories of Experiment E, are, in so far as distance is concerned, wholly differences of mental time-form and are not at all differences of "mental mass."

§ 61. This being so, it is incumbent upon our hypothesis to show how the heft of the pencil could affect the distance-series. I think it does so in two different ways. First, the skin and the flesh beneath, being flexible, the harder the pencil is pressed the more is its influence spread peripherally, both by pressure and by tension, and, consequently, the larger is the stretch of skin actually stimulated. Second, it is not unfair to suppose that the more intense any stimulation is, the longer may its results continue, centrally, —continue, either by penetrating through a longer series of cells, or, by more prolonged activity of the same cells. Thus, we see how a part of the heft of the pencil can be transformed into "length" of resulting

distance-perceptions, without committing the mistake (which we have called the "root of error" in much modern psycho-physical experimentation) of taking for granted that, either the intensity, or the mass of the resultant feeling, or of the ultimate central physical activity, need to bear any *constant* ratio whatever to the intensity or amount of the peripheral stimulation.[1]

I am inclined to account for the over-estimation of the "heavy" stimulations of Experiment E by the two above explanations, rather than by saying that we get any notion of longer distance by reason of the greater "volume," or "mass," or the greater intensity of feeling which, undoubtedly, results from the harder pressure of the pencil.[2]

[1] As no one has, to my knowledge, ever claimed that mere intensity formed a part of the mental constituency of space, I need not discuss such a proposition here. Possibly, intensity is but serial relationship between time-form and mass. If Professor Charles Pierce is correct, then the physical atoms, and all kinetic phenomena, are but different time-manifestations of that absolute continuum, in which all physical and psychical things find ultimate identity.

[2] Professor James, in his theories of space and distance, makes great use of "The Feeling of Crude Extensity," . . . "the original sensation out of which all the exact knowledge of space . . . is woven" (James' Psychology, II, 134, 135). The doctrine I advocate holds that our every notion of extensity is wholly an expression of time-extension. If "crude" and "original" mean "independently of time-form," then I should say that, independent of time-form, there could be no "feeling of extensity," while, theoretically, there might still be "big feelings" and "little feelings," in the sense of "more feeling" and "less feeling." Of

EXPERIMENT G.—WITH A SINGLE PIN.

§ 62. Our Law Three has been based upon the fact that when, for a given stimulation, no specific habit has been sufficiently developed to react, under all conditions, with exactness, then the influences of a lot of

course, without any "mass," or content, at all, there would be no feeling. And if crude mass, or content, of any kind, is what Professor James means by "original sensation," why, then, every mental state is woven out of such, but, as much so, our emotional states, and all other "states," as our knowledge of space. Yet, I am inclined to think that this man of genius has, as usual, laid hands on an important truth. While I deny that, "*all* our exact knowledge of space" is woven out of this "mass"-element of mental variability, yet I believe that *a part* of the difference between different space-perceptions may be a difference in their respective mass-elements. It is true that a "big sound" *may* form a part of our perception of a big space; but it is also true that a "small" light may form a part of our perception of a vastly larger space, as of a distant star. What I doubt, in criticism of Professor James's theories, is, that my spatial perceptions of a star are "*all*" woven out of the "mass"-element of mental variability, or even any important part of them. Professor James says, of number, that "all would be one big, buzzing, blooming confusion," till the first shock of succession occurred, and, that, out of that, or in it, would the feeling of duality arise. Precisely similarly of distance. I say there might be big buzzings and little buzzings, big blooms and little blooms, but never would any feeling of distance arise till some feeling had buzzed or bloomed *so long a time*. That buzzing or blooming "so long," would be the original distance-event; and, whatever consolidated, or modified, "memory"-event might, subsequently, rise as the resultant representation of that new feeling, would be wholly proportional to, and expressive of, the "so long" of the original event, and in no way proportional to its bigness or its littleness.

allied but looser tendencies are awakened to fill out the resulting perception. Hence, "over-estimations." The question now arises: What would happen, if a lot of these allied tendencies should be awakened by other means than through the uncertainty of the outer stimulations alone, as, for instance, through strongly allied inner associations, passing in the current of our thoughts at the moment the outer stimulations were made? Do our passing thoughts, also, modify our perceptions under Law Three?

§ 63. The following experiment gives us very curious information on this point, enabling us to connect, in an instructive manner, our foregoing experiments and thesis regarding outer Perceptions, with the more common doctrines regarding inner Associations.

In proper holders, prepare several sets of medium-sized sewing-needles, two in each set, in such ways that the distance between the points, in each set, shall be regularly graded, from 1 mm. up to 8 or 10 cm. apart. Also prepare one "control" holder, bearing a single pin of medium size and fineness of point. It is best, at the outset, to show these needles (not the "control") to the person to be experimented upon, and to make sure that he is visually appreciative of the exceeding minuteness of the minimal distance. Until the experiment is concluded, the subject should never be permitted to suspect that there is any "control" holder, containing a single pin.

Choose, now, any part of the body, the tip of the tongue, for several reasons, being preferable to begin with. Select, at first, such a pair of needles that their two points will, with certainty, be perceived double when applied to the region of skin chosen. Continue to apply the points, stepping them about, here and there; sometimes pressing both, and, again, only one, till the subject is perfectly familiar, not only with the prick of the single points, but also with the two as "felt together." Then proceed likewise with the series of points, using each pair in order, down toward those of the minimal distance apart.

As soon as the subject becomes uncertain in his judgments, begin to explain to him how much concentration and attention and practice will do toward increasing his power of discrimination. Perhaps it is well here to read to him, from Ladd's text-book (Phy. Psy., page 411), that Goldscheider could, on the finger, feel two points only 2 mm. apart, and actually to show him a pair of needles with points separated by that distance. Then begin experimenting again, and, just as the judgments grow uncertain, introduce the "control" pin, unknown to the subject, and keep him well up in the effort of discrimination. Having performed this experiment on many persons, I have never yet failed, with proper care in introducing the "control," to procure, in time, a series of successive judgments which, when once begun,

may be continued indefinitely, and in all of which the single pin-prick is clearly and uniformly perceived as double.

§ 64. Whatever its relation to the outer object and to nerve-ends outwardly stimulated, the mental state itself is unmistakably real and distinct; the two points are as sharply felt separate as ever any two points can be, and continuously and evenly so, for so long as the pins remain applied. The pins may be held in the subject's own grasp, and rocked back and forth, causing now one point to be felt, and now the other; and so with all sorts of varying intensity, consequent upon "feeling around" with them,—all precisely as if there really were two pins.

§ 65. The line of direction, between the two points, may also be discriminated, with reference to the topography of the region of skin worked upon, and to surrounding objects, as definitely as any perception of direction ever is. If the conditions of application remain unchanged, and in the absence of any conscious suggestion, either from any outward circumstance, or even as prompted from within one's own imagination, then the seeming line of direction is likely to remain, to the subject, unchanged also. Yet the apparent direction is, plainly, not one of passing accident. Nearly every region has a most constant direction, one strongest developed and most natural to rise; a sort of "local

sign," dependent upon the contour and local expression of each particular region. As, for instance, on the trunk and limbs, in the absence of all suggestion to the contrary, the direction is more likely to appear longitudinal than transverse. The more highly discriminative the region is spatially, the less constant is the native direction likely to be. Upon the tongue, the direction is likely to change whimsically, upon the slightest movement given to the pin. The fingers, though as delicately sensitive as the face, are less constant in this native direction. The slightest passing suggestion, however, is likely to establish a corresponding notion of direction for the points. If the subject, just as he closes his eyes, sees you raise toward his forehead a holder really having two pins so set that when applied they would naturally fall vertically, and you then deftly substitute the "control" without disturbing his expectations, the line of direction is pretty sure to be felt vertically. Thereafter, the direction will seem to change by reason of the most unconscious suggestions. If the subject take the holder in his own hand, and keeping his elbow as nearly in one point as possible, step the pin along naturally from the center of the forehead around and down toward the ear, the direction will constantly move to correspond to the radius of the arm in its several positions. The changing of the experimenter's arm will also change the direction accordingly. In short, the

direction once established for the immediately subsequent applications, it will usually appear to follow all the changes which it ought to follow, were two real pins, rigidly fixed with reference to each other, applied by precisely similar movements.

§ 66. More surprising still is the apparent distance between the two fictitious points. This is different, in the absence of positive suggestions to the contrary, for each particular region of the body; and it appears pretty nearly to correspond to the distance given by Weber as the minimum at which two compass points are perceived double for the same region of skin.

For instance, the subject having been brought to feel the "control pin" double upon the tip of the tongue, if then he be asked what the distance seems, his judgment is likely to be in the neighborhood of 3 mm. If, however, the pin be stepped slowly along the side, or rim, of the tongue, the distance will spontaneously, and without any suggestion of any such occurrence, seem to him to spread to four, five, or perhaps more, millimeters apart. Upon the middle upper surface of the tongue, the distance is likely to seem 2 cm. In short, whenever the pin is stepped from place to place over parts having different native space-values, the apparent distance between the points changes spontaneously in accordance with these native values. And all this will happen, though the experiments be so managed, as were

our own, that the subject shall all the time be under the impression that compass points have been set at varying distances, which he is to estimate in order to test the accuracy of his judgments. This "spreading" phenomenon is particularly marked upon the forehead, where, throughout, the "native distances" are pronouncedly irregular. In the centre of the forehead, the distance is likely to seem one or one and a half centimeters; stepped along horizontally to a point above the tip of the ear, it will spread till it appears like three or four centimeters.

That there is a natively developed constant-distance which, in the absence of all passing suggestion, spontaneously rises to shove itself in between the two points, when, by proper management, these have been psychologically separated for the subject; and that this constant is definite for each particular region of skin, but different for different regions of skin, is a fact which almost any one, by proper care, can with great ease demonstrate to himself. From the results of a tolerably large number of tests upon four persons, and of less numerous tests upon ten other persons, I got the following averages for these "native distance-constants" of the regions specified. I must state, however, that the average error for these figures was considerable, and, altogether, I have not had the opportunity, as yet, to determine the matter for the several regions of the

body as fully as the importance and significance of the phenomenon seems to promise would be profitable:

TABLE OF NATURAL DISTANCES.

Tip of tongue	3 mm.
Side of tongue	5 "
Top of tongue, middle	10 "
Rim of lower lip, middle	4 "
Rim of lower lip, side	5 "
Forehead, middle	10 "
Forehead, side	20 "
Scalp, over ear	30 "
Ball of forefinger	4 "
Back of hand	10 to 15 "
Forearm	15 to 25 "
Abdomen	35 to 50 "

§ 67. According to the above method of procedure, the subject is to be kept in entire ignorance that a single, or "control" pin is ever being used upon him. This, I think, is advisable, if not absolutely necessary, for every first demonstration of the experiment upon a novice. But after a person has once been operated upon in this manner, and has himself experienced how perfectly real and distinct the illusions are, he may then successfully operate upon himself with equal satisfaction. The vividness of the results improve rapidly with practice, until it becomes quite possible for one to carry the phenomenon in an unbroken series from the lips to the finger tips, and thence, backward and downward across the abdomen and thighs, to the points of

the toes, the distance between the pins seeming to vary the while from a few millimeters to as many centimeters, according to the native distance-value of the region traversed.

And what we particularly call attention to is, that the distance apart at which the subject begins to be uncertain of his ability to perceive the pins separately, is approximately the distance which, for the same region, they appear apart when fictitiously presented by the "control" pin. Another way of stating this is, that Weber's distances are approximately our native distances. We shall, however, soon come to look upon Weber's distances as less constant functions, and of far different significance, than has been commonly conceived in the abundant literature devoted to them.

Not only may two fictitious points be perceived from the single pin, but upon the tongue and lips at least three, four and even five can be felt. And these may be felt as if arranged somtimes in lines, sometimes in triangles, and sometimes in parallelograms or squares.

Such being the phenomena and the method of their experimentation, we may now seek their explanation and their significance under our general thesis.

§ 68. Every perception is the resultant of a sum of tendencies, partly peripheral, and partly associative or central. In ordinary perception, we are apt to underrate both the amount and the character of the portion filled

in by the central influences. Before me now is a curious picture. When I look at it expecting to see a certain face of an old woman, I see it as vividly as in an ordinary portrait. When I look expecting a certain girlish face, I see that equally well. The image thrown on the retina is the same in the two cases. The portions filled in centrally are very different. In proportion to their difference, we must appreciate how great was the amount of the central contribution to each perception. Also, realizing how all parts of the perception appear equally *vivid*, objective and real, we are reminded that there is no difference to be distinguished in *kind* or *quality* between the portions which are due directly to peripheral excitation and those which are filled in through associative, or central excitation.

What is requisite in such cases of two possible perceptions for the same outward stimulation, seems to be, that they both shall be of such a nature that their peripheral elements are identical. A certain drawing may be perceived as two straight lines, or as a cross, but never as a circle. The peripheral elements for the first two of these perceptions would be identical; they would not be identical for the last two.

When a pin is pressed upon the flexible skin ever so lightly, a group of several nerve-fibers is pretty sure to receive stimulation. What sort of perception will follow, will not depend upon the number of nerve-ends,

nor upon their peripheral grouping—whether in one bunch or in two—but upon the portion filled in or not, or centrally. The "feeling of double" originates from a shock of succession. But once originated, it may be revived and filled in centrally, precisely like any other centrally contributed portion of any perception. If the same set of nerves from whose successive stimulation a "feeling of double" originated, is thereafter simultaneously stimulated, a perception of two points may result therefrom. Yet, we have to note that such will not necessarily result. What we have thereafter is the possibility of two different perceptions from the same peripheral stimulation. Apparently, the peripheral elements are the same in both. If the definite central influence acts jointly with the definite peripheral influence, we perceive two points. If the peripheral influence acts alone, we perceive one point. The question of perceiving two points or one is, therefore, the question of the proper central influence being or not being brought into activity conjointly with the proper peripheral influence, and without regard to how it shall be awakened.

In our experiment with the single pin, it is not difficult to see how the proper central influence is awakened. We are expecting to feel two points at precisely the place touched by the single pin. If the same central parts are active in expectation which are active in

realization,—and, from modern psychology, it is fair to suppose they are,—then we may say that all the central influences requisite to a perception of two points at the place in question, are already in partial activity through the expectation. Consequently, when the proper peripheral influence is aroused by the pressure of the pin, all the conditions requisite to the perception of two pins are fulfilled. The proper sensational tang and strength of feeling is furnished through the peripheral influence, and the doubleness is achieved through the filling in of the proper central influence. The whole thing is possible, through the congruity of the peripheral elements furnished by the passing outward stimulation with the peripheral elements demanded by the expected perception. Were they not alike, they would either inhibit each other or compromise, by calling up some resultant perception different from either.

In short, therefore, the reason that we do not perceive a pin-point double is, because the group of nerves so stimulated has not of itself sufficient associative connection with the proper central influence to call it up jointly. The reason we do perceive the pin double in our experiment is, because the expectation provides the proper central influence. The reasons we cannot double up any and every sort of perception, by expecting to perceive it double under outward conditions that commonly present it to us single, are of two kinds. First, if the

perception is complicated, the peripheral elements actually furnished from without, are the less likely to agree with the elements furnished from within the expectation; *i.e.*, the more difficult is it for the expectation and the realization to agree in those elements wherein they must agree. And, second, the more complicated the perception, the more difficult is it for us *to expect it* to be doubled. If only one operator were present, it would be impossible for the subject to expect two right hands to be laid on his breast at once. Only very simple perceptions, similar to those which frequently are perceived double, could ever be successfully manipulated by the method of our present experiment.

§ 69. The distance shoved in between each pair of our fictitious points is now easily to be explained. Having explained how, through the expectation of the subject, the proper central influence was aroused requisite to the perception of doubleness, I need only to say, now, that the fact that some sort of distance was also perceived under the same circumstances is to be explained in a similar way. It remains to make plain how it happens that a certain definite length of distance is called up constantly for each particular region, and a different length for different regions.

This, after our many discussions, is not difficult. Every point of our skin has during life been knit up into many serial stimulations, and, proportionally, has

become joined to as many different distance-tendencies. If the habits of reaction have not been narrowed to some particular length, then, upon stimulation, come in all the results of uncertainty, so abundantly formulated by us under Law Three. The distance phenomena of our Experiment G are but extreme instances of such uncertainty. The subject expects some sort of distance. He is not precisely sure how the needles, or the compass points, will be set. Upon application of the "control" pin, his mind ranges up and down within the limits of his expectation, precisely as upon the application of any piece of apparatus from Experiment A. Presently, as the resultant of the sum of all the tendencies so awakened, both from the pin and from the expecting, a definite perception rises, precisely as in any other judgment of distance. The only things remarkable about such occurrences are the weakness and uncertainty of the peripheral tendencies awakened by the single pin, and the unusual strength of the central tendencies awakened by the intensity of the expectation. To some, it may not yet seem quite clear why the fictitious distance should be so different for different regions.[1] They vary for precisely the reason

[1] If our Experiment G distances vary considerably from those given by Weber, I am inclined to think this is chiefly due to the fact that we used pin-points, and he used rounded points of a millimeter in diameter.

that Weber's distances vary, namely, because, through life, the distance experiences of the different regions have been different. In turn, this, partly, is due to *gross* anatomy, and partly to the exigencies of life. A long line *can* be drawn on the leg; not on the tongue.

§ 70. It should be plain now, I think, why, when we expect the proper distance in our G experiment, we perceive it fictitiously. It only remains to make clear why, when we expect any other distance than the proper one, the "control" pin will give us no perception of distance whatever.[1] For instance: Why, on the abdomen, if we expect two points, 3 mm. apart, do we get no perception? Or, why, on the arm, if we expect 3 cm., do we get nothing? To answer this: we think in various terms, sometimes in visual images, sometimes in muscular images. No doubt, dermal images enter into all of our "certain" judgments in the foregoing experiments. All we have learned about development of central influences and tendencies through peripheral experiences implies this. "Uncertainty" implies imperfect development of dermal images. We can *visualize* a nine-pin square resting on our arm at any time with accuracy. We can perceive it *dermally* with uncertainty; and I am inclined to believe that we are less able to *expect* it in any given

[1] Of course, I speak approximately. Both the proper distance and the expected distance have a short, permissible range.

place, in dermal images, than we are to *perceive* it, in terms of dermal feeling, in the same place.

This being so, I think the answer to our above questions is, that when we expect any sub-threshold distance for any region of skin, we do not expect it in dermal images, but only in visual or muscular images. Consequently, no dermal tendencies are aroused, such as are requisite to coöperate with the peripheral stimulation to produce a dermal perception. Indeed, I suspect that the reason we cannot discriminate the sub-Weber distances is just because, on account of their not being sufficiently developed, we cannot expect them or conceive them in proper dermal terms. When we expect the proper distances, we can conceive this in dermal terms, and, therefore, the perception is possible. When we expect too long a distance, here, also, the dermal images are lacking; for the range of distance-tendencies, developed with any sort of permanent central efficiency for any given limited area of skin, is, I suspect, very limited. What we get in our fictitious G perceptions seems to be a general average, or resultant, of this rather narrow range of habitual distance-reactions for the given very limited area on and immediately around which the pin is pressed, or expected to be pressed. And, of course, this is constant for a particular region and different for different regions.

EDUCATION OF ARTIFICIAL SPACE-RELATIONS.

(*Experiment H.—Preliminary Report.*)

§ 71. Having conducted the following experiment upon only one person, the results are not yet well enough determined for full publication. Yet, their nature is already such as to warrant mention of them among our other experiments, to which they stand in close relation, and which, from the first, they were designed to supplement.

§ 72. The apparatus may be described roughly, as follows: Conceive 49 cylindrical keys, each 5 mm. in diameter, arranged in a square, 7 keys on a side. The keys are kept raised by springs, and when one is pressed down, a pin or needle, screwed in the lower end of the cylinder, is pressed against the skin. The distance from center to center of each pin is 1 cm., measured in lines parallel to the sides of the square. The whole is secured in a proper box or frame. The peculiarity about the instrument concerns the one cylinder in the center of the square. This has no pin in its end, but is attached to a lever 4 cm. long, which runs 1 cm. beyond the outer limits of the square formed by the 49 keys. In the end of this lever is a pin, which, when the center key is pressed down, is forced against the skin, similarly as are the other pins. The center key, working

the lever, must be so adjusted that it shall feel to the subject "to press down" in a manner not noticeably different from the other keys; the crucial thing being that *he* shall not suspect that the pin it presses on the skin is not in the end of the cylinder directly under its center, precisely as are all the other pins. To this end, also, the box is made so as to conceal its inner arrangement. The top shows only the heads of the keys numbered with plain figures in regular order. The bottom is a perforated board, on a level just below the end of the pins as they rest without being pressed down. The ends of the pins are, therefore, not seen if the box be raised and examined; and as the bottom is perforated all over, and not alone for the pins just in the square, the subject, with proper care, is little likely to suspect the one thing that *must* be kept from him, namely, the true position of the "decoy" pin worked by the center key. The box is carried by an adjustable arm and stand that permits it to be lowered and held over any region of skin, as near as may be desired, without resting upon it.

Various "dies" are now made by cutting holes out of heavy pasteboard squares, or notches out of strips, to the end that, by use of them, any desired combination of keys may be pressed down at once.

§ 73. The intention of the experiment is to cultivate artificial space-relations by continued education upon

regions of the skin, which have a high degree of native sensitiveness or capability of dermal discrimination, but which, from reasons of their location and "lot" in life, have received but very meager spatial education and perception development. The notion is that, in such regions, one's natural spatial discrimination will at first be unable to distinguish the true position of the decoy pin; its education will depend upon the actual serial relations artificially formulated by the experiments ; in these series the decoy pin is made to fall into the same time series that it naturally would fall into were it situated in the center of the square instead of outside of it. Consequently, to the subject, the feeling of this pin gets woven into his newly-educated space-nature in precisely the same relations as would have happened were the decoy pin really in the middle of the square, where the subject from the first has supposed it to be.

We selected for our first experiment the abdomen, which, besides filling the above requirements, enabled the subject to see and operate the apparatus. The experiment was continued ten or fifteen minutes for nearly three months. I explained to my subject that my purpose was to test the limit to which one's power of dermal perception or discrimination could be educated. The plan was to begin with so many keys or pins that the subject could not possibly discriminate the

true position of the decoy pin, or, in fact, of any other single pin. It will be understood that this was not difficult to do, since the distance apart, here, at which two compass points can be distinguished separately is, according to Weber, from 5 to 7 cm. We then proceeded to drop out certain pins, or rows of pins, always working toward distincter masses and combinations. At the end of a week I began using full rows of pins, never fewer, at that time. The subject would work a while, pressing the keys himself, and trying to fasten in his mind just how the several rows individually felt. He would then close his eyes and I would press the keys of one row and then another, asking him each time to identify the row he felt.

At the end of two weeks I began dropping out pins from the single rows, and combinations of pins in the same row. This was kept up till the summer vacation began, when we were compelled to suspend the experiment. But such progress had been made that finally I was able to press successively, in any order, the two end and the middle (our decoy) pin in any row, and the subject correctly locate the keys that had been pressed, and the order in which he had felt them; yet do so without the least suspicion that the middle or decoy pin was outside of the square altogether, and 3 cm. distant from where he felt it to be, namely, right under the center of the key that pressed it down. At times, also,

I made tests like the following: Pressing one end key and the middle key of any row, I would then press the other end key of the same row, and ask if the last pin was "between," or outside the two former ones. Or, again press the two end keys and then the middle key, and ask the same question. Though the subject would not always give the correct answer, he would do so in more than 80 per cent of the cases, which is, perhaps, as much as he would have done had the decoy pin really been in the center where he conceived it to be.

The significance of this experiment in the light of the doctrine of the Genetic origin of our spatial perception, is obvious. Artificially, we seem to have actually demonstrated here what the Genetic Theory asserts theoretically: that the absolute fixity which appears to characterize the spatial relations of one point to another, and of one line and surface to another in those objective perceptions of places and things which we call "*real*," is wholly dependent upon a certain definite fixity of *time-order* or "*time-relation*" in our original experiences, and which is preserved, with greater or less integrity, in our habits of mental reproduction.

GENERAL SURVEY AND SUMMARY.

We have now examined the results of our several experiments, and discussed many questions. Other points of importance remain to be considered under a comparative survey of the several tables. But I must be content to knit these up with a general survey of the whole field. I will, therefore, close the present paper with a summary of our thesis as it now stands:

1. Certain parts of the brain react correspondingly to certain definite combinations of peripheral nerves.

2. Every combination (peripheral or central) acting together once, develops to that degree a tendency to act together again; the more frequently the parts act together the stronger is this tendency.

3. This law holds good, within certain limits, as well of successive combinations or series, as of simultaneous combinations.

4. Successive combination influences, in a marvelous and wholly unexplainable manner, the simultaneous activities of the same parts.

5. Until successive stimulation of the combination occurs, the mental correspondent of the simultaneous stimulation of it will be one homogeneous qualitative whole.

6. Subsequent to proper stimulation of the total combination successively in definite parts, the mental result of the simultaneous stimulation of the whole will present a collection of parts so arranged that the successive qualitative wholes which followed each other in definite order in the original succession, will be represented in a simultaneous picture or *presentation* of them, by corresponding qualitative parts.

7. In the simultaneous presentation each infinitesimal part bears a qualitative nature that can, by the psychologist, be traced back to a greater or less dependence upon the quality of a certain corresponding term in the original succession.

8. Also all these parts are arranged in a manner respectively expressive of the positions which their corresponding terms occupied in the time-order of the original succession.

9. If the terms of the original succession were sufficiently broken in order, or unlike in quality, a *numerical* presentation will result from the subsequent simultaneous stimulation.

10. If the original series was continuous a *distance* presentation is given — as a line.

11. If the original was continuous, but its terms unlike qualitatively, the presentation will be both numerical and spatial; as a line of bead-like intensities, or of different colors.

12. Dermal presentations are chiefly founded upon original terms alike in kind (not different as are colors) but differing in intensities.

13. Numerical and distance presentations are not governed alone by the last preceding successive stimulations of their corresponding sets of peripheral nerves, but,

14. Since all the past serial modes in some degree modify the simultaneous reaction, and this at any one time can present but *one* serial arrangement, the latter becomes an averaged resultant of all the serial modes which the given peripheral combination has experienced throughout life.

15. If any peripheral combination through life has been stimulated sufficiently more together than separately, the presentation will be a numerical "whole." As pin-point and small dermal areas.

16. If the nerves are strung out in a line, and the line is stimulated through life more serially than all together, the presentation will be a lineal "distance presentation."

17. The distance presentation, or the apparent length of the line will express the average length of all the time series in which the peripheral line has through life been stimulated.

18. Upon simultaneous stimulation of any dermal surface, there tend to rise into presentation simulta-

neously, all the distance series developed between every possible pair of points in that surface; the consequent presentation is a specific resultant of the sum of all those particular tendencies; such we call a "surface" presentation.

19. Since each separate tendency that is active at any moment modifies and is modified by every other tendency active at that moment, the apparent distances, in the resultant presentation, are modified according to the particular peripheral arrangements combined in stimulation at that moment.

20. All distance and spatial perceptions are partly dependent upon the particular peripheral arrangements to whose simultaneous stimulations they respond; and

21. Partly upon the average sort of distance-series experienced through life between each pair of points in these arrangements.

22. The time-length of the serial stimulations, received on any particular region of the body, will depend partly upon the contour of that region; and

23. Partly on the outer events to which that region is customarily subjected.

24. Lines two feet long can be drawn on the leg; such cannot be drawn on the tongue. Every nerve in any region is likely to enter into every length of combination possible to that region. Every length of combination has proportionate influence on all subsequent

tendencies in which that nerve, by life's developments, is involved. Consequently, all the distance presentations of the leg are likely to appear relatively longer than those of the tongue, or of other small members. On the other hand, since short lines can be drawn on any member, large or small, and, since any line, long or short, may be drawn at any rate of motion, quick or slow, therefore, the average distance-tendencies developed for any particular region will depend chiefly upon the outer events of that region.

25. Partly, the distance tendencies will depend on the length of lines most commonly drawn on a given region; and

26. Partly, on the rates of motion most common in the drawing.

27. For various stretches of skin not too distant from each other, or covering members not too unlike in function, the average length of lines drawn are likely to be approximately the same, and so, also, the average time-rates for the two stretches. Hence, the presentations of different lengths of peripheral lines are approximately proportional to their absolute lengths.

28. For regions distant from each other (as forehead and abdomen), or covering members of unlike function (as tongue and cheek), the lines drawn, and the rates, are likely to average unequally. Hence, the respective presentations of equal peripheral lines

upon the two regions, commonly appear of different lengths.

From all of the above, the following laws may be deduced, relative to different regions.

29. The larger the region, the longer will its units of peripheral distance appear to be in spatial presentations.

30. On any region longer in one direction than in another, the units of the longer direction will appear longer than those of the other.

31. The longer the motions commonly made by or on any member, the longer will its presentations appear. (Those of forearm are longer, relatively, than those of upper-arm.)

32. Same for direction. (Thus, forearm makes wider sweeps radially than forward and backward, like a piston. As our experiments show that the distances across are seemingly greater than along the forearm, we may presume that the advantage of wider sweep overcomes, here, the disadvantage of less anatomical length.)

33. The more rapid the movements made by or on any member, the shorter its presentations appear.

34. Same for direction.

35. Gravity being a force continually tending to lengthen movements over the skin in a downward direction, thus making the average downward experiences of

life longer than others, therefore, vertical presentations are apt to appear longer than horizontal ones.

36. The more frequent any particular length or rate of movement on any region, the more do all the presentations of that region approximate to that length.

37. Same for direction.

So far, I have discussed only continuous lines and stretches of skin. I will now consider portions of skin *not* continuous.

38. If any two separate points be simultaneously stimulated, the presentation will express the average of all serial combinations in which those two points, through life, have participated. (Same as Law Fourteen, for continuous lines or areas.)

39. Since an infinite number of curved lines of varied length can and, through life, are likely to be drawn between every pair of points, therefore they become knit up into an infinite number of tendencies.

40. The averages of these, for different pairs of points on similar regions, will be, respectively, proportional to the corresponding right-line distances between those points.

41. Laws Six, Seven and Eight hold good under the supposition that the present stimulation affects all terms of the peripheral combination with equal intensity. If it affects some terms with greater intensity than others, corresponding effects of intensity will appear in the proper terms of the presentation.

42. Consequently, in presentations from separated points, the terms correspondent to those points appear with increased intensity.

43. The more frequently any given separated presentation is awakened by proper peripheral stimulation, the more marked, through the modifying influence of each repetition, becomes the difference of intensity between the correspondent, or stimulated terms and the remaining terms, *i.e.*, the more distinct and differentiated becomes the presentation.

I esteem Laws Forty-one, Forty-two and Forty-three to be of the highest importance in all psychological science. From them, chiefly, arise the highly differentiated and clear mental states which we call perceptions of definite objects and things. They largely account for the differences of clearness, strength and coherency between sensations and ideas; between perceptions and conceptions.

44. The more distinct and differentiated the separate terms of a presentation become, the more distinctly do they become numerical presentations.

45. To formulate the genesis of numerical presentations, we must, therefore, determine the laws governing the simultaneous and successive combinations of separate points. This may be done as follows:

46. The further points are apart, the more likely they are to be stimulated separately.

47. The more they are stimulated separately, the more distinct and accurate are their corresponding numerical percepts.

48. The further points are apart, the more distinct and accurate are their numerical percepts. (This is our old Law Three.)

49. The more frequently any x number of separate points are stimulated in the same combination, successively or together, the more distinctly and accurately will these presentations be of x numbers.

50. In any fixed area of skin, the greater the number of points in any given combination, the less frequently through life are they likely to be stimulated together, either simultaneously or successively.

51. Therefore, the greater the number of points in any given distance, the less accurate and clear those numerical perceptions.

52. Or, for any given category of number, the greater the distance, the more certain and clear the numerical perceptions. (Our old Law Two, for Number.)

53. Since stimulation of two separate peripheral points, *A B*, tends to recall the distance tendency corresponding to the right-line, *AB*; and since the stimulation of any other points on the line, *AB*, simultaneously with the stimulation of *A B*, tends, also, to recall this same right-line tendency; therefore, on any

given peripheral right-line, the greater the number of separate points stimulated simultaneously, the more accurately will the right-line presentations arise—(commonly we would say, the more accurately will it be judged).

54. Since the right-line presentation is the shortest presentation, the greater the number of points, the shorter will the line appear. (Our old Law Four, for Distance.)

55. Of the presentations of the same absolute distance, those from a continuous straight-edge will appear shorter than those from separated points. (This law is for simultaneous stimulation. The reverse may hold good for successive.)

56. Other things being equal, the further apart two points are, the less likely is a line to be drawn between them.

57. Since the accuracy of a distance presentation depends on the frequency of its proper stimulation, therefore, from the above, other things equal, the greater the distance, the less accurate should be its presentation.

58. On the other hand, since the simultaneous stimulation of separate points calls up the straight-line tendency developed between them, and, by so doing, strengthens that tendency; therefore, other things being equal, the greater the distance, the *more* accurate

should be its presentation. This is but saying: The more distinctly separated our notions of local points are, the more accurate our distance-judgments of them. (Our old Law Two, for Distance.)

59. By reason of the conflict of above Laws Forty-two and Forty-three, and because many varied local conditions, formulated by our several above laws together, govern the sort of distance-series most common to each particular region; therefore, the absolute distance most correctly judged, for each particular region, must be determined empirically, and, when so determined, is the resultant expression of all these conditions.

60. When so determined for any given region, it will hold good for that region, that all distances, longer or shorter than this preferable distance, will be presented with proportionally less accuracy than the preferable distance. Consequently—

61. For distances above the preferable distance, the greater the distance the less accurate the presentations. And—

62. For distances below the preferable distance, the greater the distance the more accurate the presentations. (This goes with our Law Two of the Distance-judgments in our various experiments.)

63. All the above laws, both of Number and of Distance, hold good in their general intent,—*i.e.*, in

so far as the actual result is not modified by new conditions—in the more complex presentations of two-dimensioned space.

64. Since each distance tendency awakened, at any given time, tends, in its measure, to modify all the other distance tendencies active at that time; therefore, if we attempt to judge any particular element or line of distance, in any given peripheral combination, the consequent percept, or judgment, will be, in some degree, modified from what it would be, were that element stimulated by itself, and will be modified proportionally to the mean of all the distance tendencies developed between every possible pair of points in the *total* peripheral combination. This I will hereafter speak of as the Law of Average Distances.

65. Hence, of the distance-judgments of lineal figures, those of equilateral triangles are less than those of circles; those of circles less than those of squares; and those of squares less than judgments of single lines presented independently of all figured implications. (The basis of comparison is assumed here to be the sides of the right-line figures and the diameters of the circles.)

66. For similar reasons, the greater the number of internal points, in any figured arrangement of points, the shorter will be the distance presentations. (Those of the four-pin triangle, of our Experiment B, will

appear shorter than those of the three-pin triangle, etc.)

67. Since (Law Fifty-two) the greater the distance, the more accurate the numerical presentations, and since, in two-dimension figures, the distance tendencies modify each other proportionally to the average distances; therefore, the numerical presentations of separate points, set in figured arrangements, will be accurate and clear in proportion to the average distance of all the points. (Four points in a square will be more accurately presented than four in a triangle, or four in a line, etc.)

68. Owing to the facts — (*a*) That different parts of the brain act on different occasions; (*b*) That parts which have acted together serially tend to repeat these series; (*c*) That by varied peripheral experiences, the several central parts have been combined into innumerable different series; (*d*) That every passing presentation being vastly complex, the sum of tendencies of which it is the resultant must at all times contain terms which are members of some certain series having yet remaining terms uncompleted in their passing, and, therefore, at the given moment having yet a certain remnant of tendency toward continuance; (*e*) And that proper nourishing processes renew the parts to approximately constant inclination to activity; therefore, the vast complexity of centrally developed serial tendencies

once under way, they continue to combine and to recombine indefinitely and independently of freshly incoming peripheral impulses.

69. The content of thought at any given moment expresses the sum of two separate sources of influence: the already progressing central processes and the incoming peripheral influences.

70. Both processes play ultimately upon the same brain parts and upon the same general storehouse of developed tendencies. The tendencies that at one moment are active in the central stream, are identically the same that at another moment, or it may be at the same moment, are played upon by the incoming peripheral stream; those of both are of the same general origin and nature, and governed by the same laws. Therefore: all the foregoing laws of combination and development, apply as well to the central stream as to the peripheral stream, and as well to the joint combination of the two streams as to either.

71. Every passing presentation expresses the resultant of the sum of the joint streams of tendencies—the peripheral and the central—under mutual modification of education; those of the one working to modify those of the other equally with those of its own.

72. According to the methods of judging peripheral combinations used in the experiments of this paper, the subject knew the certain range of presentation and

categories which he could expect. Consequently, from *the* expecting of these,—from the ranging of the imagination over the possible categories to be applied,—a certain range of tendencies corresponding to the range of experimentation were already awakened into activity, preparatory to the application of the peripheral category.

73. Also, the concentration of the attention upon a particular region awakens, in a vaguer degree, all the categories common to that region.

74. Consequently, upon application of any single piece of apparatus, three distinct sets of categories were awakened, from the joint action of which the final perception was an expression; namely (1) The tendencies developed for the special peripheral combination actually applied. (2) The range of tendencies expected and governed by the range of experiment. (3) The possible range of tendencies developed for this particular region.

75. The presentation resulting from these three sets of tendencies will be governed, first, proportionally by their respective strengths — and second,

76. Proportionally by their mutual congruity.

77. Of the three sets, the influence of the peripheral tendencies is likely to be the strongest, those of the expected categories the next strongest, and the local possible categories the weakest; and for the following

reasons: The application of the apparatus by external means continues the influence of one single definite combination unchanged through a relatively prolonged time. On the other hand, owing to the continued "ranging" of the expectations, the influence of each particular expected category is less sustained, and also by the modification of mutual successions, becomes weaker and less explicit. The possible categories are even more flittingly and vaguely awakened. Moreover, the peripheral tendencies form a definite combination *in the habit of holding together* for a prolonged time, while the expected categories not only *do* shift and range in perpetually changing combinations, but *never have had any* fixed order of combination, such as is prescribed to them by the range of the experiment and by the vast variety of the passing stream of present events.

78. Therefore: the consequent presentation is likely to be moulded most by the form of the peripheral combination; next in amount by the expected range of categories set by the experiment; and least by the possible categories set by the character of the particular region worked on.

79. The fewer times any given peripheral combination has occurred, the less strong will it be, and the more will the consequent presentation express the average of the several other combinations into which,

through life, the separate terms of the given combination have severally entered.

80. Therefore: the weaker or less developed the given peripheral combination on any region, the more will the presentation drift toward an expression of the average of all the possible categories of that particular region. (First form of our Law Three.)

81. If the applied category is below the average of total categories developed for the given region, the drift of error will be toward "over-estimation"; if above, toward under-estimation.

82. Since the weaker the peripheral influence, the stronger proportionally is the central influence, therefore: the less developed or "more uncertain" the peripheral combination, the more will the presentation drift to an average of the expected categories.

83. Since the shorter the distance the more "uncertain" the presentations, therefore: the shorter the distance the greater the over-estimation, both by reason of 81 and of 82. (This is our old Law Three, of Number and of Distance.)

84. From 81, therefore: in general, small numbers and short distances are over-estimated; large numbers and long distances, under-estimated.

85. Since we call those presentations the most "accurate" which we receive through our highest developed region or sense, and since our several dermal

regions are variously, and some of them very inferiorly, developed; therefore: for many regions what are the most constant and certain judgments for that region may not be what we commonly conceive of as the most "accurate" judgments, even from that region. Also,

86. Since the drift due to the inaccuracy from the category actually applied to a given region may compensate the whole general drift of the region in comparison with the judgments of our highest developed senses, therefore: we often call judgments "accurate" which are functionally very inaccurate for the region and categories really involved. (By these compensations are explained the phenomenon, so common throughout all our experiments, of apparently increasing accuracy, through those categories, where, by all our most general laws, there ought to be displayed an ever-increasing inaccuracy. As for instance, in all the shorter distance categories of Experiment A, where we discovered constantly increasing numbers of correct judgments with increasing numbers of pins, although really the greater the number of pins the more difficult and uncertain the dermal presentations ought to be.)

87. Since the different regions and members of our body vary greatly in contour, size, function and lot in life's experience, therefore and consequent to the above: the whole general scale of presentation for different regions and members vary greatly one from another.

88. Our most mobile regions are the most used; therefore, the most highly developed; therefore, their scales of judgments are deemed the most accurate.

89. As a matter of fact, our highest developed regions are not only the most mobile, but also are both our smallest and our quickest-moving members—(eye, in proportion to number of nerves stimulated, tongue and fingers).

90. Hence, our most accurate scales of judgment are "short scales" of judgment, relatively to the scales of our larger and slower moving regions and members.

91. Hence, so-called "over-estimation" is the common trait of all our larger, clumsier and less developed dermal regions. (Hence, the over-estimations common to all the tables, and in our several experiments for the various regions worked upon, the eye outscaling them all.)

92. Hence, also, the relative order of accuracy, maintained alike throughout all the several kinds of judgments, whether of number, of distance, of figure or otherwise, which is observed between the different regions of the body. (Shown in "Amounts of Average Error," "Number of Correct Judgments," "Over-estimation," etc., of our tables, and confirming the following order for the parts we worked on: Tongue, Forehead, Forearm, Abdomen.)

93. Hence, also, the relative accuracy of judgments

made in different directions upon the same general region.

94. Not only do the foregoing laws hold good for the judgments of simultaneous presentations, but also they hold, within limits, for judgments of serial stimulations (such as from a pencil drawn on the skin).

95. Therefore, and since every distance presentation is in essence an expression of serial time-form: The particular time-rate of movement over a given peripheral distance will express itself in the resultant presentations.

96. Hence, quick movements seem shorter, and slow movements seem longer.

97. Hence, and because heavy peripheral pressure spreads to affect actually longer stretches of skin, and perhaps, as well, because the greater intensity of the resulting nervous currents may cause the central reaction to endure longer, or to spread further through the central parts, therefore: The heft of moving peripheral stimulations governs, in some degree, the resultant presentations.

98. Hence, heavy movements seem longer, and light ones shorter.

99. The foregoing laws also manifest other modifications under more complicated modes of procedure appropriate to the differences in method involved. As for instance:

100. Where, in applying certain lines and figures, "rocking" is allowed, in imitation and revival of the original successive events in which our notion of such figures originate, the presentations are appropriately more accurate than where the rocking is carefully excluded.

101. It may be observed throughout our several tables, and best in the Summary Tables, that the accuracy of presentation, as shown in the Number of Correct Judgments and in the Amounts of Error, is relatively greater for Distance that for Number, and for Figure than for Distance.

102. Also, the Number-judgments vary more in accuracy, as between the several regions of the body, than do the Distance-judgments, and the Distance-judgments than do the Figure-judgments. Our thesis is now able to offer explanations of these things as follows:

103. The several terms of the original dermal series, from which dermal presentations both numerical and spatial develop, are of identical *quality*, and, to a very large degree, of like *intensity* throughout each given event. Consequently, that numerical separateness, which characterizes our topographical notions of any dermal region, and which is fundamentally based upon difference in intensity between the terms of the developed "memory series" of each particular region, is

almost wholly dependent, for its origin and development, upon the laws of simultaneous stimulation of separate points.—Stimulation of the separate points calls up the distance tendency of the whole right-line of intermediate points, but emphasizes the two end points by the passing stimulation of them, while the intermediate points are not so emphasized; hence, the difference of intensity, which is transmitted and augmented from repetition to repetition of the particular occurrence. Hence, it will be observed that, in *dermal* presentations, the numerical development is dependent upon previous distance development; *consequently, for the most part they remain inferiorly and less accurately developed.* It is instructive to note that in hearing the conditions are wholly different. The difference in quality of the many tones is a rich basis in itself for the development of numerical separateness; and there is little if any development based upon the spatial arrangements of the nerve-ends. Consequently, our auditory numerical judgments are highly developed and accurate, while we are capable of comparatively little strictly auditory spatial discrimination. In the eye we have rich, qualitative and spatial basis for perception. Consequently, visual perceptions are highly developed, both numerically and spatially.

104. The numerical range of possible combinations on any given region is practically unlimited, while the distance range is much determined by the local contour

and dimensions of any particular region. Any number of taps can be made successively on the tongue, but lines more than two or three inches long cannot be drawn on the tongue. Consequently, the possibilities of error are always greater for numerical than for distance presentation, and betray themselves more under the relatively different degrees of development evolved between the several regions of the body.

105. Our figure-judgments (I do not say presentations) are based both upon numerical and upon distance presentations ; they may take their cue from either and from many other suggestions in operation in simpler judgments but too complicated for the limits of this paper.

106. Hence, judgments of certain familiar figures, as of the various triangles, squares and circles in our experiments, are throughout more accurate than the judgments of the very numerical and distance *presentations* upon which the figure-*judgments* are based.

107. Whether we judge our more complex figure presentations as wholes — as when judging if a given impression is of a triangle or of a square — or confine our judgment to regard some particular element of the total presentation — as when judging the length of some one side of a triangle — the results give evidence of conformity to the same fundamental laws which have governed the development of presentations from the beginning.

108. The integrity of our general thesis within the realm of the ideational or representative processes, as well as in presentative ones more directly correspondent to outer impressions, is also upheld experimentally throughout our work.

109. Finally, by such methods as that of Experiment H, we are able artificially to construct genuine inner perceptions of space and spatial relations in strict conformity with the hypothetical laws which we have deduced for the origin and development of all spatial perceptions naturally.

Looking backward, we may now summarize our thesis as follows:

Its origin and foundation must be fundamentally placed in the following law: Presentations of Number, of Distance, and of all Spatial Figures and arrangements in general, are alike based, primarily, upon serial events differing greatly in mode, such as become characteristic of those modes of presentation which we call numerical, extential and spatial, but all of them governed by the same fundamental laws of relationship. By reason of this, *all simultaneous presentations are dependent upon, and expressive of, the several modes of serial occurrence out of which, through life, they have evolved, and become differentiated.*

From the simplest presentations to the most highly developed functions of judgment, we find this same system of laws articulated everywhere into one common Genetic System of Mental Development.

Presenting these conclusions, I now beg the reader to turn back and to re-read the Introduction of this paper, the better to recall its full purpose, and again to orient its whole subject with Psychological Science in general.